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STUDY OF PLANE WAVE
IMPINGEMENT ON A THIN PLATE
CAPABLE OF DEFORMATION

by

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December, 1991

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Study of Plane Wave Impingement on a Thin Plate Capable of Deformation

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ABSTRACT

This thesis models the effects in a fluid medium of a plane wave that has impinged upon a reinforced plate. The wave equation for pressure, and the equation of a thin plate combined with other equations are coupled at the interface between the fluid and the thin plate. The actual modeling is accomplished in a fortran computer program written to run on the Naval Postgraduate School's main frame computer. The program uses extensive finite differencing on a domain, assumed to be a small section of an infinite interface between the fluid and the plate, to simulate the deflection of the thin plate and the pressure disturbances in the fluid medium. To accomplish this, each of the above equations will be scaled or nondimensionalized. Additional finite differencing is explained which covers the special cases for the side boundaries of the fluid domain, and the artificial boundary created to model infinity. Different beam spacing is explored for its effect on the magnitude of the propagating modes of the scattered pressure wave.

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I. INTRODUCTION

This thesis models numerically, the effects of a pressure disturbance incident on a fluid loaded reinforced plate using finite difference methods. The program produced explores the reaction of a plane wave striking the plate (hull of a ship) at different frequencies, for different plate lengths and for various spacings of the reinforcing beams (or members).

Figure 1 shows the problem being modeled. The figure shows a cross section of a reinforced flat plate with water on the top side of it and a vacuum below it. For this problem, it is assumed that the plate is infinite in the x direction along the horizontal axis. Since the plate structure is periodic in space, the model will only look at one section of this infinite plate. The behavior at other sections is assumed to be the same. We will use three fundamental equations in the model. The first equation is the wave equation, the second is the linear-inviscid-force equation, and the third is the equation of motion for a thin plate. The coupling of these equations at the fluid-plate interface makes the problem very difficult to solve analytically.

Chapter II derives the relationships among these three equations as they are used in the model. Assumptions used in the program are spelled out in this chapter. Chapter II also includes the steps necessary to scale each equation to facilitate its use in the computer program.

Chapter III explains the operation of the FORTRAN computer program used to model the problem. The program takes as an input the incident and specularly-reflected sound waves and determines the scattered pressure due to the fluid-structure coupling. To make best use of the Naval Postgraduate School's mainframe computer the main program is divided into a number of smaller subroutines. These are also described in Chapter III.

Chapter IV discusses the results of the program runs. It also reviews the weaknesses in the model and model limitations.

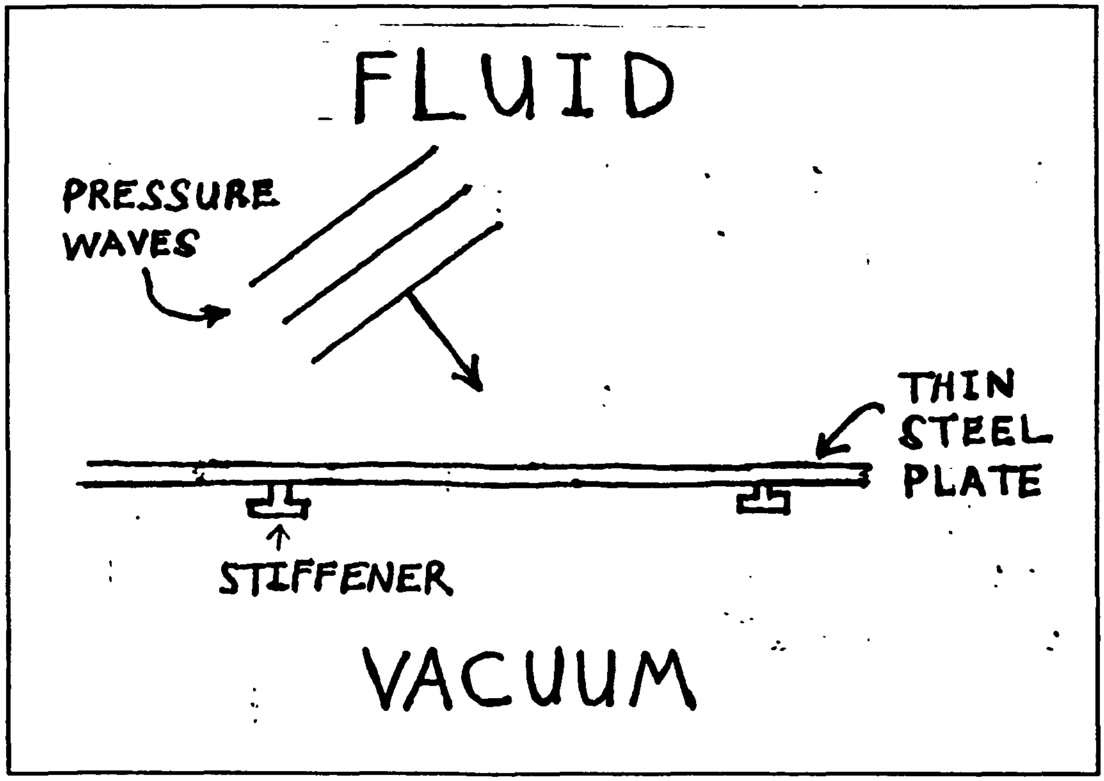


Figure 1. Cross Section of Hull

II. THEORY

A. THE WAVE EQUATION

It is known that sound travels through seawater, or any fluid medium, as a pressure wave. This is possible because seawater is a compressible fluid which is affected by pressure disturbances. To approximate the movement of sound in a fluid medium we have used the linearized, lossless wave equation for pressure as derived in Kinsler, Frey, Coppens and Sanders [Ref. 1:p. 104],

$$\nabla^2 p^T = \frac{1}{c_f^2}\frac{\partial^2 p^T}{\partial t^2}, \tag{2.1}$$

where c_f is the fluid sound speed and p^T is the total pressure.

The pressure wave can be split up into various parts, each of which is a solution of the linear wave equation. In this case we want to be able to solve for pressures that would be present in the fluid which modify the pressure resulting from specular reflection of an incident plane wave from an acoustically hard surface. This known specularly reflected pressure wave is combined with the computed scattered pressure wave to make up the total reflected pressure wave.

Symbolically, we have:

$$p^T = p^I + p^R + p^S, \tag{2.2}$$

where p^T is the total pressure, p^I is the incident pressure, p^R is the specularly reflected pressure, and p^S is the scattered pressure. The total radiated pressure is a superposition of the reflected and scattered pressures.

To eliminate the specularly reflected wave from the equations of motion we apply the boundary condition

$$\frac{\partial p^I}{\partial y} + \frac{\partial p^R}{\partial y}\bigg|_{y=0} = 0. \tag{2.3}$$

In this case the normal derivatives of the incident and specularly reflected waves must cancel at the surface of the plate. The vertical coordinate Y is equal to zero at the surface.

We assume that the source of the incident pressure is far from the surface so that a plane wave approximation is adequate to model the incident pressure, p^I.

The time harmonic incident and specularly reflected plane waves are of the form:

$$p^I(x,y,t) = e^{i\omega\left(\frac{\bar{x}\cdot\hat{d}^I}{c_f} - t\right)} \tag{2.4}$$

and

$$p^{R}(x,y,t) = e^{i\omega\left(\frac{\vec{x}\cdot\hat{d}^{R}}{c_f} - t\right)}, \tag{2.5}$$

where

$$\hat{d}^{I} = \begin{bmatrix} \cos\theta_I \\ -\sin\theta_I \end{bmatrix} \tag{2.6}$$

and

$$\hat{d}^{R} = \begin{bmatrix} \cos\theta_I \\ \sin\theta_I \end{bmatrix}. \tag{2.7}$$

are the unit direction vectors of the two plane waves.

Both the incident wave, Equation 2.4 and the reflected wave, Equation 2.5 must satisfy Equation 2.1. Therefore, because of superposition, p^s must also satisfy the wave equation,

$$\nabla^2 p^{S} = \frac{1}{c_f^2}\frac{\partial^2 p^{S}}{\partial t^2}. \tag{2.8}$$

B. THE LINEAR INVISCID FORCE EQUATION

The linear inviscid force equation, taken from Kinsler, Frey, Coppens and Sanders [Ref. 1:p. 104], is basically the inviscid Navier Stokes equation in which nonlinear terms

have been neglected. It is valid for acoustic disturbances of small amplitude, and can be written

$$\rho_f \frac{\partial \vec{u}}{\partial t} = -\nabla p^T, \tag{2.9}$$

where ρ_f is the constant equilibrium density of the fluid, and $\vec{u}$ is the velocity of the fluid.

Since we assume no cavitation of the fluid or penetration of the fluid at the fluid-plate interface, the component of the fluid velocity normal to the interface and the plate transverse velocity are equal at the fluid-plate interface at all times t. Letting w represent the transverse displacement of the plate (y direction), we have

$$\rho_f \frac{\partial^2 w}{\partial t^2} = -\frac{\partial p^T}{\partial y} \qquad at \quad y=0. \tag{2.10}$$

Through use of Equations 2.2 and 2.3 the total pressure p^T can be replaced by the scattered pressure p^S:

$$\rho_f \frac{\partial^2 w}{\partial t^2} = -\frac{\partial p^S}{\partial y} \qquad at \quad y=0. \tag{2.11}$$

C. EQUATION OF MOTION FOR A THIN PLATE

To properly understand what is happening on and within the thin plate, a reference system must be assigned. In Figure 2 we see that the vertical plane will be designated the y-axis in which the thin plate experiences transverse displacement w. The horizontal plane will be the x-axis which has longitudinal displacement u. To set up our equations,

we assume that the centerline (axis) of the beam stays perpendicular to its cross section during bending.

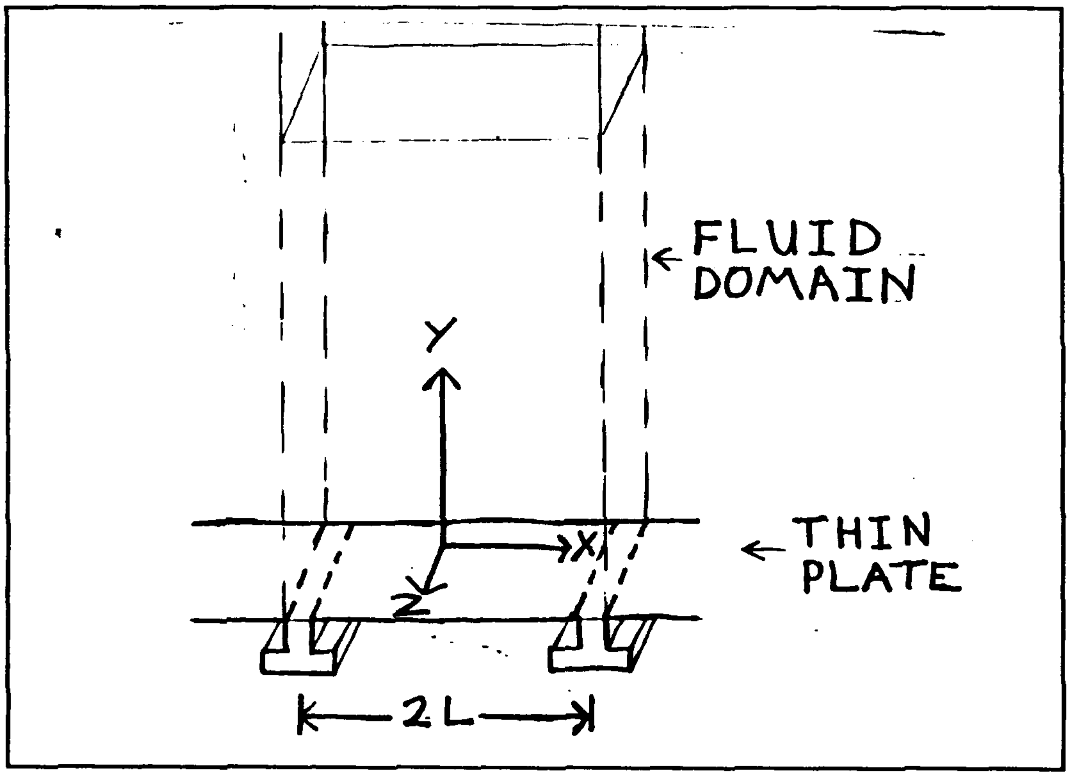


Figure 2 Plate Reference System

As put forth by Kinsler, Frey, Coppens and Sanders [Ref. 1:p. 58], Hooke's law states that if the strain is small the stress is proportional to it. The stress on the plate is described by the equation

$$\sigma_{xx} = -y \frac{E}{1-\nu^2} \frac{\partial^2 w}{\partial x^2}, \tag{2.12}$$

where:

σ_{xx} is the stress on the plate, E is the Young's modulus, and ν is the Poisson's ratio.

Consider a small section of the plate of area $\Delta x \Delta z$ with moments and forces applied as shown in Figure 3:

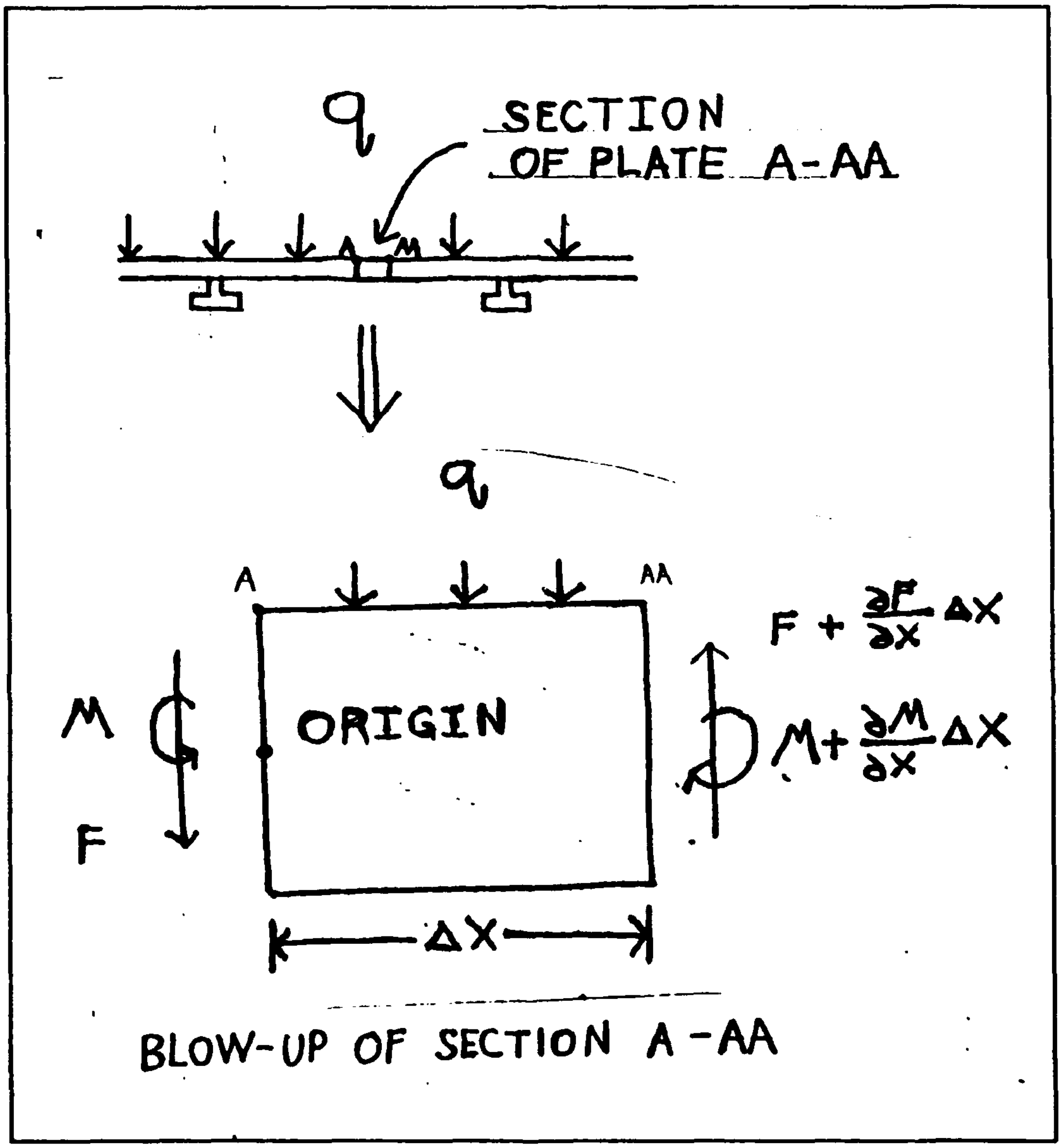


Figure 3. Forces and Moments

Note that there are no forces or moments applied which have z dependence. The motion therefore is assumed to be two-dimensional. Since the plate is not rotating, a force balance is performed on the moments and moment arm, neglecting terms of order

Δx^2, giving us

$$\frac{\partial M}{\partial x} = F, \tag{2.13}$$

where:

M = bending moment on the element,

q = load on the plate element (uniform for a normally incident wave),

and F = flexural force applied to the element.

Equating forces in the y-direction applied to the section of plate yields

$$\rho_s \frac{\partial^2 w}{\partial t^2}(h\Delta x\Delta z) = \frac{\partial F}{\partial x}\Delta x\Delta z - q\Delta x\Delta z, \tag{2.14}$$

where:

S = cross sectional area of the plate,

ρ_s = density of beam section,

h = plate thickness.

Dividing out the $\Delta x\Delta z$ terms yields

$$-q + \frac{\partial F}{\partial x} = \rho_s h \frac{\partial^2 w}{\partial t^2}. \tag{2.15}$$

Using the definition of the moment

$$M = \int_{-\frac{h}{2}}^{\frac{h}{2}} y\sigma_{xx}\,dy \approx -\frac{EI}{1-\nu^2}\frac{\partial^2 w}{\partial x^2}, \tag{2.16}$$

where the moment of inertia is found from

$$I = \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 dy = \frac{h^3}{12}, \tag{2.17}$$

we obtain the equation for deflection of a plate

$$\frac{EI}{1-\nu^2}\frac{\partial^4 w}{\partial x^4} + \rho_s h \frac{\partial^2 w}{\partial t^2} = -q. \tag{2.18}$$

The loading of the thin plate is due to pressures exerted by a fluid in contact with the thin plate. Rewriting Equation 2.18 including the pressure terms yields:

$$D\frac{\partial^4 w}{\partial x^4} + \rho_s h \frac{\partial^2 w}{\partial t^2} = -p^T, \tag{2.19}$$

in which

$$D = \frac{Eh^3}{12(1-\nu^2)}, \tag{2.20}$$

is the flexural rigidity and h is the plate thickness, Junger and Feit [Ref. 2:p. 194].

D. THE SCALED EQUATIONS

1. The Scaling Factors

In order to use the three governing Equations 2.1, 2.9 and 2.19 in the finite difference code, they must be scaled or nondimensionalized. The steps are as follows. The independent variables $\bar{x}$ and t are scaled through use of the fundamental length L and the wave frequency ω, respectively.

$$\bar{x} = \frac{x}{L}, \quad and \quad \bar{y} = \frac{y}{L}, \tag{2.21}$$

where $\bar{x}$ and $\bar{y}$ are the scaled spatial coordinates, and L is the one half the distance between stiffeners measured in the x-direction.

The equation for the scaled time is

$$\bar{t} = \omega t, \tag{2.22}$$

where ω is the frequency of excitation of the incident plane wave.

We then have

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial(L\bar{x})} = \frac{1}{L}\frac{\partial}{\partial\bar{x}}, \tag{2.23}$$

and

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial(\frac{\bar{t}}{\omega})} = \omega\frac{\partial}{\partial\bar{t}}. \tag{2.24}$$

Additionally, the dependent variables w and p must be scaled as follows. For the displacement of the plate (in the y-direction) we put

$$\bar{w} = \frac{w}{\hat{W}}, \tag{2.25}$$

where $\bar{w}$ is the scaled vertical displacement of the plate and $\dot{W}$ is the displacement scaling factor. For pressure, put

$$\bar{p}=\frac{p^s}{L\omega^2\rho_f\hat{W}}=\frac{p^s}{P_o}, \tag{2.26}$$

where $\overline{p}$ is the scaled pressure and $P_o = L\omega^2\rho_f\dot{W}$ is the pressure scaling factor.

The wave numbers for waves propagating in the fluid, and along the plate are given as follows,

$$k_f=\frac{\omega}{c_f}, \tag{2.27}$$

and

$$k_p=[\frac{\omega^2\rho_s h}{D}]^{\frac{1}{4}}, \tag{2.28}$$

respectively.

Additional parameters resulting from the scaling analysis are

$$\Omega=\frac{k_f^2}{k_p^2}=\frac{\omega}{\omega_G}, \tag{2.29}$$

where Ω is the squared ratio of fluid to plate wave numbers and ω_G, the coincidence frequency of the plate, is defined by

$$\omega_G=[\frac{\rho_s h c_f^4}{D}]^{\frac{1}{2}}. \tag{2.30}$$

We also define the dimensionless quantity

$$\epsilon = \frac{\rho_f c_f}{\rho_s h w_G}, \tag{2.31}$$

where ϵ is a dimensionless quality which is a measure of the fluid loading on the plate.

2. The Wave Equation

The wave equation governing the scattered fluid pressure was found to be:

$$\nabla^2 p^s = \frac{1}{c_f^2}\frac{\partial^2}{\partial t^2} p^s. \tag{2.32}$$

Scaling the above equation yields:

$$\omega^2 \frac{P_o}{c_f^2}\bar{p}_{\bar{t}\bar{t}} = \frac{1}{L^2}P_o\bar{p}_{\bar{x}\bar{x}} + \frac{1}{L^2}P_o\bar{p}_{\bar{y}\bar{y}}, \tag{2.33}$$

which reduces to the desired non-dimensional form,

$$\bar{p}_{\bar{x}\bar{x}} + \bar{p}_{\bar{y}\bar{y}} = \frac{L^2\omega^2}{c_f^2}\bar{p}_{\bar{t}\bar{t}} = L^2 k_f^2 \bar{p}_{\bar{t}\bar{t}}. \tag{2.34}$$

3. The Linear Inviscid Force Equation

The linear inviscid force equation

$$\rho_f \frac{\partial^2 w}{\partial t^2} = -\frac{\partial p^s}{\partial y} \tag{2.36}$$

in scaled form reduces to

$$\bar{w}_{\bar{t}\bar{t}} = -\bar{p}_{\bar{y}}. \tag{2.37}$$

4. Thin Plate Equation

The dimensional equation of flexural rigidity is:

$$D\frac{\partial^4 w}{\partial x^4}+\rho_s h\frac{\partial^2 w}{\partial t^2}=-p^T|_{y=0}. \tag{2.38}$$

By substituting in the scaling factors it becomes:

$$D\hat{w}\bar{w}_{\bar{x}\bar{x}\bar{x}\bar{x}}+\rho_s h\,\omega^2L^4\hat{w}\bar{w}_{\bar{t}\bar{t}}=-\bar{p}^TL^5\omega^2\rho_f\hat{w}, \tag{2.39}$$

or, after more simplification, it becomes

$$\bar{w}_{\bar{x}\bar{x}\bar{x}\bar{x}}+k_p^4L^4w_{\bar{\tau}\bar{\tau}}=-\frac{L^5\epsilon k_f k_p^4}{\Omega}(\bar{p}+2\bar{p}^I). \tag{2.40}$$

This is the scaled form of the thin plate equation that will be used in the computer program. Note that at y = 0, p^R equals p^I. Hence, $\overline{p}^T$ has been replaced with $\overline{p}$ + $2\bar{p}^I$.

5. Stiffeners

In the present analysis the stiffeners will be treated in an idealized way. The end conditions we will assume are imposed at the point where the stiffeners are located will be equivalent to those of a clamped plate for which the boundary conditions are w = 0 at $\bar{x} = \pm 1$, and $w_x = 0$ $\bar{x} = \pm 1$.

E. THE FLUID DOMAIN

As stated before, it is assumed the plate on which the sound wave impinges is infinite. That assumption makes it possible to establish the following conditions. The boundary conditions on the section of plate we are looking at will be periodic. That is

to say, for normal incidence of p^I, displacement on the plate at $\bar{x} = -1$ will be the same as at $\bar{x} = 1$. For arbitrary angles of incidence, the pressures at $\bar{x} = \pm 1$ differ only by a scaling factor determined by the angle of incidence. Figure 4 and the following equations will show how the time harmonic solution is dealt with in the problem.

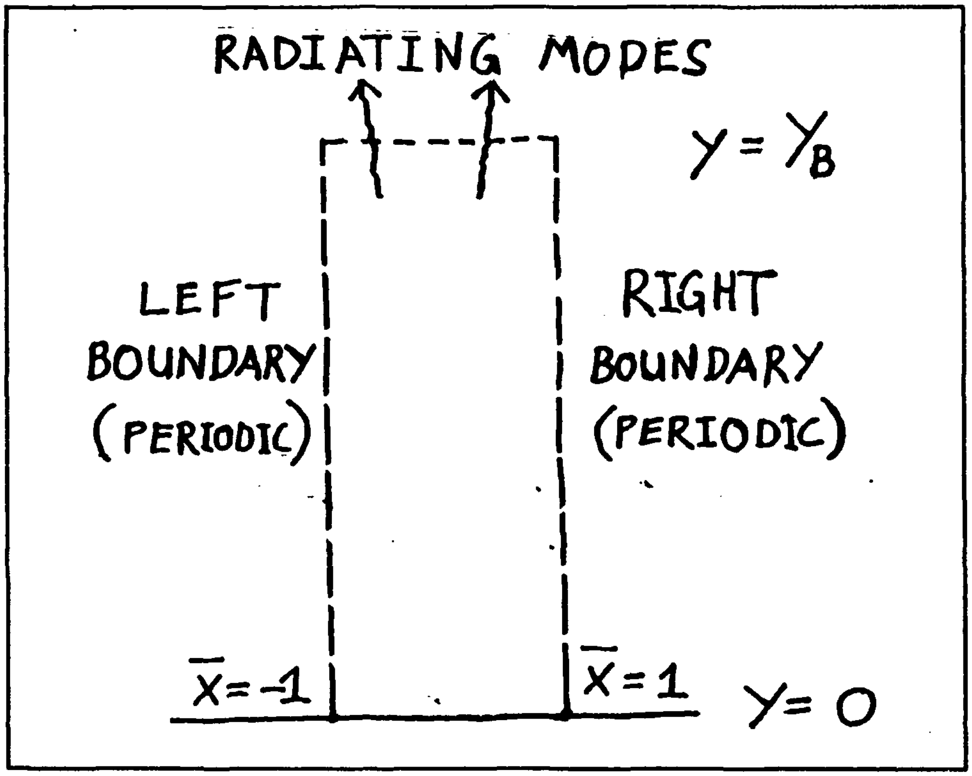


Figure 4. Side View of Fluid Domain

To understand how the fluid pressure is disturbed, the wave equation is considered for which a prescribed pressure is applied from the fluid to the surface of the plate. This uncoupled problem can be solved analytically by the method of separation of variables.

The scaled wave equation is

$$P_{xx}+P_{yy}=\alpha^2 P_{tt}. \tag{2.41}$$

Applying periodic boundary conditions results in

$$p(-1,\bar{y})=p(1,\bar{y}) \quad \text{and} \quad p_x(-1,\bar{y})=p_x(1,\bar{y}). \tag{2.42}$$

The forced pressure at the plate surface is given by the boundary conditions

$$p(\bar{x},0,t)=g(x)e^{-i\bar{t}} \quad \text{where} \quad g(-1)=g(1) \quad \text{and} \quad g_x(-1)=g_x(1), \tag{2.43}$$

where:

$g(\bar{x})$ = prescribed pressure applied to the fluid by the plate.

Assuming a time harmonic solution with equivalent frequency dependence to that of the forcing function, the scattered pressure can be expressed as

$$p(\bar{x},\bar{y},\bar{t})=\phi(\bar{x},\bar{y})e^{-i\bar{t}}. \tag{2.44}$$

By substitution of Equation 2.44 into Equation 2.41 we obtain the reduced Hehmholtz equation for $\phi(\bar{x},\bar{y})$,

$$\phi_{\bar{x}\bar{x}}+\phi_{\bar{y}\bar{y}}+\alpha^2\phi=0, \tag{2.45}$$

with boundary conditions:

$$\phi(-1,\bar{y})=\phi(1,\bar{y}) \quad \text{and} \quad \phi_x(-1,\bar{y})=\phi_x(1,\bar{y}) \tag{2.46}$$

and

$$\phi(\bar{x},0) = g(\bar{x}). \tag{2.47}$$

Henceforth we will be dropping the bars from all equations, giving us the following,

$$\phi_{xx}+\phi_{yy}+\alpha^2\phi=0, \tag{2.48}$$

$$\phi(-1,y)=\phi(1,y), \tag{2.49}$$

$$\phi_x(-1,y)=\phi_x(1,y), \tag{2.50}$$

$$\phi_x(-1,y)=\phi_x(1y). \tag{2.51}$$

To solve for ϕ the standard method of "Separation of Variables" is applied to the equation. First we write ϕ as a product of functions of one variable each,

$$\phi=X(x)Y(y). \tag{2.52}$$

Upon substitution into the Helmholtz equation and separation of variables, we find a solution

$$X_n(x)=C_n e^{in\pi x}. \tag{2.53}$$

When solved for Y, two answers result:

$$Y_n(y)=Ae^{iy\sqrt{\alpha^2-n^2\pi^2}} \quad \text{when} \quad \alpha>n\pi, \tag{2.54}$$

and

$$Y_n(y) = Ae^{-y\sqrt{n^2\pi^2-\alpha^2}} \quad \text{when} \quad \alpha < n\pi . \tag{2.55}$$

These solutions are chosen to avoid the physically implausible solution that the scattered pressure as y approaches infinity will blow up represent an incoming wave.

Combining our solutions we have

$$\Phi = \sum_{n=0}^{\infty} \phi_n(x,y) = \sum_{n=0}^{N} \alpha_n e^{in\pi x + i(\sqrt{\alpha^2 - n^2\pi^2})y} + \sum_{n=N+1}^{\infty} \alpha_n e^{in\pi x - y\sqrt{n^2\pi^2-\alpha^2}}. \tag{2.56}$$

Note that as y approaches infinity the decaying terms become exponentially small. Now from our boundary condition at y = 0, α_n can be found such that

$$g(x) = \sum_{n=0}^{\infty} \alpha_n e^{in\pi x} \quad \text{with} \quad \alpha_n = \frac{1}{2}\int_{-1}^{1} g(x) e^{-in\pi x} dx. \tag{2.57}$$

It is important to note that in the numerical solution of the fully coupled problem, the radiation waves must be properly modeled as y approaches infinity. The numerical domain cannot be semi-infinite in extent in the y direction. It must be truncated at some value of y, which we call y_B, where a boundary condition must be imposed to ensure proper modeling of the radiated pressure.

F. THE PERIODICITY PROBLEM

To ensure that a wave with any angle of incidence can be correctly modeled by the computer program the periodicity must be worked out. If we were to have a wave normally incident on the plate we would expect a solution for the pressure which is periodic in x to be

$$\phi = \sum_{n=0}^{\infty} \phi_n(y) e^{in\pi x}. \tag{2.58}$$

To account for an arbitrary angle of incidence, Equation 2.58 needs to be modified in the following manner. A correction factor which takes into account the phase change at x = -1 vice x = 1 must be added. The modified equation for the scattered pressure is,

$$\phi = \{\sum_{n=0}^{\infty} \phi_n(y) e^{in\pi x}\} e^{-ik\cos\theta_I x}, \tag{2.59}$$

where θ_I is the angle of propagation of the incident pressure wave.

Continuing with this argument one can write

$$\gamma_n = -k\cos\theta_I + n\pi. \tag{2.60}$$

Substitution of this x dependence into the Helmholtz equation yields

$$\frac{d^2\phi_n}{dy^2} + (k^2 - \gamma_n^2)\phi_n = 0. \tag{2.61}$$

Substituting these relationships back into our equation for ϕ gives us

$$\phi = \sum_{n=-M}^{N} \alpha_n e^{i\gamma_n x + i\beta_n y} + (\sum_{-\infty}^{-M-1} + \sum_{N+1}^{\infty}) \alpha_n e^{i\gamma_n x - \beta_n y}. \tag{2.62}$$

where

$$\beta_n = \begin{cases} \sqrt{k^2-\gamma_n^2} & k>\gamma_n \\ \sqrt{\gamma_n^2-k^2} & k<\gamma_n \end{cases} \tag{2.63}$$

and in which the first sum includes all values of n for which $k > \gamma_n$ and the latter two sums account for the remaining modes.

G. FINITE-DIFFERENCE MODELING

1. The Taylor Series

Refer to Numerical Solution of Partial Differential Equations: Finite Difference Methods, Chapter I, by G. D. Smith [Ref. 3] for a more complete explanation of finite difference methods.

All three governing equations will be center differenced with truncation errors of order $(\Delta x^2, \Delta t^2)$.

Finite difference approximations for u_x, u_{xx}, u_{xxxx} are

$$(u_x)_i \sim \frac{u_{i+1}-u_{i-1}}{2h}+O(h^2), \tag{2.64}$$

$$(u_{xx})_i \sim \frac{u_{i+1}-2u_i+u_{i-1}}{h^2}+O(h^2) \tag{2.65}$$

and

$$(u_{xxxx})_i \sim \frac{(u_{i+2}+u_{i-2})-4(u_{i+1}+u_{i-1})+6(u_i)}{h^4}+O(h^2), \tag{2.65}$$

where $u_i = u(x_i)$.

2. Differencing of the Wave Equation

We have already shown that the wave equation is of the following form

$$P_{xx} + P_{yy} = k_f^2 L^2 P_{tt}, \tag{2.67}$$

and that we have the following conditions,

$$x_i = i\Delta x, \qquad y_j = j\Delta y, \qquad t_n = n\Delta t, \tag{2.68}$$

where i, j, and n are integers. This allows us to describe the pressure as follows,

$$P_{i,j}^n = P(x_i, y_j, t_n) \qquad \text{and} \qquad \Delta x = \Delta y = h. \tag{2.69}$$

So by substituting into Equation 2.66 we arrive at the final form,

$$P_{i,j}^{n+1} = \frac{\Delta t^2}{k_f^2 L^2 h^2}[P_{i+1,j}^n + P_{i-1,j}^n - 4P_{i,j}^n + P_{i,j+1}^n + P_{i,j-1}^n] + 2P_{i,j}^n - P_{i,j}^{n-1}. \tag{2.70}$$

3. Finite Differencing of the Linear Inviscid Force Equation

We have already shown that the linear inviscid force equation is of the following form

$$-w_{tt} = \frac{\partial p}{\partial y}. \tag{2.71}$$

This equation is applied at y = 0 or when j = 0 in the numerical domain. The displacement w does not depend upon y, so we introduce $W_i^n = W(x_i, t_n)$ and write the differenced form of the equation as:

$$W_i^{n+1} = -\Delta t^2 \left[\frac{P_{i,1}^n - P_{i,-1}^n}{2\Delta y} \right] + 2W_i^n - W_i^{n-1}. \tag{2.72}$$

This equation is solved for the $P^n_{i,-1}$ which is exterior to the numerical domain. It can then be substituted into the differenced form of the wave equation to solve for $P^{n+1}_{i,0}$, the pressure on the plate fluid interface at time level n+1 solving for $P^n_{i,-1}$ we have

$$P_{i,-1}^n = \frac{2\Delta y}{\Delta t^2}[W_i^{n+1} - 2W_i^n + W_i^{n-1}] + P_{i,1}^n. \tag{2.73}$$

Note that W^{n+1}_i is first needed to solve for $P^n_{i,-1}$ which means a prescribed order must be followed at the fluid plate interface. First W^{n+1}_i is found, the $P^n_{i,-1}$, and finally $P^{n+1}_{i,0}$.

4. Finite Differencing the Plate Equation

The last of the major equations that we must put in the form of finite differences is the thin plate equation. The scaled thin plate equation is:

$$W_{\bar{x}\bar{x}\bar{x}\bar{x}} + k_p^4 L^4 W_{\bar{t}\bar{t}} = \frac{-L^5 \epsilon k k_p^4}{\Omega}(\bar{p} + 2\bar{P}^I). \tag{2.74}$$

Differencing this equation results in an expression for the displacement W at time level n+1 in terms of the preceding two time levels and the incident plane scattered pressure at time level n.

$$W_i^{n+1} = \frac{\Delta t^2}{k_p^4 L^4}\left[\frac{(W_{i+2}^N + W_{i-2}^n) - 4(W_{i+1}^n + W_{i-1}^n) + 6(W_i^n)}{h^4}\right.$$
$$\left. - \frac{L^5 \epsilon k_f k_p^4}{\Omega}(p_{i,0}^n + 2p^I(x_i,0,t_n)\right] + 2W_i^n - W_i^{n-1} \tag{2.75}$$

5. Finite Differencing Side Boundaries

When differencing the wave equation at the side boundaries for the pressure we have a similar problem to that found in applying the wave equation at the fluid plate interface. The problem is that there are values of the pressure needed which are exterior to the numerical domain. Periodicity will be used as an additional condition to eliminate this problem. We have the scattered pressure in the form:

$$P_{i,j}^n = \left[\sum_{n=0}^{\infty} \phi_n(y,t) e^{in\pi x}\right] e^{-ikx\cos\theta_I} \tag{2.76}$$

What we are doing here is taking the pressure at the node i = M+1, x = Δx and replacing the pressure i = -M+1 and x = -1+Δx. From Equation 2.76 we obtain

$$P_{M+1,j}^n = \sum \phi_n(y_j, t_n) e^{in\pi(1+\Delta x)} e^{-ik(1+\Delta x)\cos\theta_I} \tag{2.77}$$

$$P_{M+1,j}^n = e^{-ik\Delta x\cos\theta_I} \sum \phi_n(y_j,t_n)(-1)^n e^{in\pi\Delta x} e^{ik\cos\theta_I} \tag{2.78}$$

and we also have:

$$P_{-M+1,j}^n = \sum_n^{\infty} \phi_n(y_j,t_n) e^{in\pi(-1+\Delta x)} e^{-ik(-1+\Delta x)\cos\theta_I} \tag{2.79}$$

Now we can make the following two statements:

$$P^n_{M+1,j}=e^{-ik\Delta x\cos\theta_I}e^{-ik\cos\theta_I}\sum \phi_n(-1)^n e^{in\pi\Delta x} \tag{2.80}$$

$$P^n_{-M+1,j}=e^{-ik\Delta x\cos\theta_I}e^{ik\cos\theta_I}\sum \phi_n(-1)^n e^{in\pi\Delta x} \tag{2.81}$$

Combining the previous statements we get:

$$P^n_{M+1,j}=e^{-2ik\cos\theta_I}P^n_{-M+1,j} \tag{2.82}$$

An equivalent argument used at the left boundary yields:

$$P^n_{-M-1,j}=e^{2ik\cos\theta_I}P^n_{M-1,j} \tag{2.83}$$

6. Finite Differencing the Radiated Boundary.

The final edge of the numerical domain is the artificial boundary which must allow free passage of radiated pressure disturbances. It must be far enough away so that the evanescent modes can be neglected. To see how the boundary condition is derived we start with the scattered pressure p^s and try to derive an analytical expression for the pressures normal direction at the boundary, as is done in Scandrett and Kriegsmann [Ref. 4]. This type of radiation boundary condition is often referred to as a non-local radiation boundary condition.

From previously we have

$$p^s(x,y,t)=e^{-it}\{\sum_{n=-M}^{N}\alpha_n e^{i\beta_n y+\gamma_n x}+ \textit{evanesent modes}\}, \tag{2.84}$$

where

$$\gamma_n = -ik\cos\theta_I + n\pi, \tag{2.85}$$

$$\beta_n = \sqrt{k^2 - \gamma_n^2} \tag{2.86}$$

and

$$\nabla^2 p^S = k^2 p_{tt}^S. \tag{2.87}$$

Additionally in our problem we have

$$k = k_f L \quad \text{and} \quad k_f = \frac{\omega}{c_f}, \tag{2.88}$$

but as y approaches infinity p^S is equivalent to only the radiated modes. We include the time dependence into the a_n coefficient such that,

$$p^S \sim \sum_{n=-M}^{N} a_n(t) e^{i(\beta_n y + \gamma_n x)}, \tag{2.89}$$

and in the following steps use orthogonality to find $a_n(t)e^{i\beta ny}$. We have

$$\int_{-1}^{1} e^{-i\gamma_m x} p^S(x,y,t)dx = \sum_{n=M}^{n} \alpha_n(t) e^{i\beta_n y} \int_{-1}^{1} e^{-i\gamma_m x} e^{i\gamma_n x} dx \tag{2.90}$$

and

$$\begin{aligned} e^{-i\gamma_m x} e^{i\gamma_n x} &= e^{ix[\gamma_n - \gamma_m]} \\ &= e^{ix[-ik\cos\theta_I + n\pi - (-ik\cos\theta_I + m\pi)]} \\ & e^{ix[n\pi - m\pi]}. \end{aligned} \tag{2.91}$$

We can therefore write

where ξ = dummy variable of integration.

$$a_m(t)e^{i\beta_m y} = \frac{1}{2}\int_{-1}^{1} e^{-i\gamma_m \xi} p^S(\xi,y,t)d\xi, \tag{2.92}$$

Noting the similarity to zero in the following equation

$$\sum_{n=-M}^{N} [\frac{\partial}{\partial y} + \beta_n \frac{\partial}{\partial t}] a_n e^{-it} e^{i(\beta_n y + \gamma_n x)} = 0. \tag{2.93}$$

Which can be rewritten as

$$\frac{\partial p^S}{\partial y} + \frac{1}{2}\sum_{n=-M}^{N} \beta_n \int_{-1}^{1} e^{i\gamma_n(x-\xi)} \frac{\partial p^S}{\partial t}(\xi,y,t)d\xi - 0. \tag{2.94}$$

Remember, these equations pertain to the artificial boundary which is modeling infinity. For this problem y is equal to y_B at the artificial boundary.

When we start to finite difference Equation 2.94, the artificial boundary and use the trapezoidal rule of integration we obtain:

$$\frac{P^m_{i,JY+1} - P^m_{i,JY-1}}{2h} + \frac{1}{4\Delta t}\sum_{n=-M}^{N} \beta_n (h \sum_{l=-IX}^{IX} \delta_l e^{i\gamma_n(x_i-\xi_l)} (P^{m+1}_{l,JY} - P^{m-1}_{l,JY})) - = 0 \tag{2.95}$$

in which we take:

$$\delta_l = \begin{cases} \frac{1}{2} & l = \pm IX \\ 1 & l \neq \pm IX. \end{cases} \tag{2.96}$$

Rewriting Equation 2.95 we have

$$\frac{P^m_{i,IY+1}-P^m_{i,IY-1}}{2h}+\frac{h}{4\Delta t}\sum_{l=IX}^{IX}\delta_l\Big[\sum_{n=-M}^{N}\beta_n e^{i\gamma_n(x_i-\xi_l)}\Big](P^{m+1}_{l,IY}-P^{m-1}_{l,IY})=0 \qquad (2.97)$$

or

$$\frac{P^m_{i,IY+1}-P^m_{i,IY-1}}{2h}+\sum_{l=-IX}^{IX}A_{i,l}(P^{m+1}_{l,IY}-P^{m-1}_{l,IY})=0 \qquad (2.98)$$

Where the square matrix A is defined as

$$A_{i,l}=\frac{2h}{4\Delta t}\delta_l\sum_{n=-M}^{N}\beta_n e^{i\gamma_n(x_i-\xi_l)}. \qquad (2.99)$$

Combining the above equations gives us:

$$P^m_{i,IY+1}-P^m_{i,IY-1}+\sum_{l=-IX}^{IX}A_{il}(P^{m+1}_{l,IY}-P^{m-1}_{l,IY})=0 \qquad (2.100)$$

From the differenced form of the wave equation applied at j = IY (the artificial boundary) we have:

$$P^{n+1}_{i,IY}=-P^{n-1}_{i,IY}+\frac{\Delta t^2}{h^2k^2}[P^n_{i+1,IY}+P^n_{i-1,IY}+P^n_{i,IY-1}+P^n_{i,IY+1}]+(2-\frac{4\Delta t^2}{h^2k^2})P^n_{i,IY} \qquad (2.101)$$

Combining Equations 2.100 and 2.101 and eliminating $P^n_{i,IY+1}$ we obtain:

$$
\begin{aligned}
P^{n+1}_{i,IY} &= -P^{n-1}_{i,IY} + \frac{\Delta t^2}{h^2k^2}[P^n_{i+1,IY} + P^n_{i-1,IY} + 2P^n_{i,IY-1}] \\
&- \frac{\Delta t^2}{k^2h^2}\sum_{l=-IX}^{IX} A_{i,l}(P^{n+1}_{l,IY} - P^{n-1}_{l,IY}) + [2 - 4\frac{\Delta t^2}{k^2h^2}]P^n_{i,IY}
\end{aligned}
\tag{2.102}
$$

Now we need to put this equation into a form which makes it possible for us to see the matrixes needed to solve the problem. First we write:

$$
\begin{aligned}
&P^{n+1}_{i,IY} + \frac{\Delta t^2}{h^2k^2}\sum_{l=-IX}^{IX} A_{i,l}P^{n+1}_{l,IY} = \\
&-P^{n-1}_{i,iy} + \frac{\Delta t^2}{h^2k^2}\sum_{l=-IX}^{IX} A_{i,l}P^{n-1}_{l,IY} + \frac{\Delta t^2}{h^2k^2}[P^n_{i+1,IY} + P^n_{i-1,IY} + 2P^n_{i,IY-1}] + \\
&(2 - 4\frac{\Delta t^2}{h^2k^2})P^n_{i,IY}
\end{aligned}
\tag{2.103}
$$

We define

$$
\delta_{i,l} = \begin{cases} 1 & if \quad i=l \\ 0 & if \quad i \neq l \end{cases}
\tag{2.104}
$$

$$
\mathrm{A}_{i,l} = \delta_{i,l} + \frac{\Delta t^2}{h^2k^2}A_{i,l}
\tag{2.105}
$$

and

$$
\mathrm{B}_{i,l} = -\delta_{i,l} + \frac{\Delta t^2}{h^2k^2}A_{i,l}
\tag{2.106}
$$

Equation 2.102 can then be written in matrix form as

$$
\mathrm{A}\vec{P}^{n+1}_{IY} = \mathrm{B}\vec{P}^{n-1}_{IY} + \vec{C}
\tag{2.107}
$$

where

$$C_i = \frac{\Delta t^2}{h^2 k^2}[P^n_{i+1,IY} + P^n_{i-1,IY} + 2P^n_{i,IY-1}] + (2 - 4\frac{\Delta t^2}{h^2 k^2})P^n_{i,IY} \tag{2.108}$$

Then the equation for the pressure vector at the next time level is given by:

$$\vec{P}^{n+1}_{IY} = A^{-1}[B\vec{P}^{n-1} + \vec{C}] \tag{2.109}$$

The pressure at $P^{n+1}_{i,IY}$ is then assigned by taking the i^{th} component of the vector $\bar{P}^{n+1}_{IY}$. In solving the matrix Equation 2.107 we use the LINPACK library Gaussian elimination programs (CGESO and CGESL).

III. DESCRIPTION OF PROGRAM

A. GENERAL

The computer program written for this thesis simulates the effects of a pressure disturbance on a fluid loaded reinforced thin plate. A detailed description of the theory behind the program can be found in Chapter II. A pressure wave in a fluid medium is moving towards a reinforced plate which is flat, and is supported by stiffeners which are at a constant spacing on the non-fluid side of the plate. The non-fluid side is assumed to be a vacuum. When the pressure sound wave hits the plate there will be some (a very small amount) distortion of the plate in the vertical direction. This displacement will cause a disturbance in the fluid medium. The scalded pressure amplitude for different frequencies and beam spacing will then be plotted and the magnitude of the propagating modes will be recorded.

The program stores the time dependent wave equation with time harmonic forcing by allowing transient effects to radiate away from the numerical domain. When a steady; state has ben achieved the code automatically stops the calculations.

B. SUBROUTINES

To make effective use of the Postgraduate School's mainframe computer, the program has been broken down into a number of smaller programs (i.e. subroutines) which can be more efficiently programmed and run.

The main program is used to coordinate the subroutines and to pass information from one subroutine to another. The following are descriptions of the subroutines.

1. Subroutine INIT

The INIT subroutine initializes the computer program. In this subroutine we set up our two main matrices. The first of these is the "P" matrix, which represents the fluid pressure in the fluid domain. The second is the "W" matrix, this represents the displacement of the plate surface which bounds the fluid medium.

Although we have the ability to change any number of parameters in the program, we will concentrate on two. These are the beam spacing, and the frequency of the pressure wave impinging on the plate. Both of these values are assigned in this subroutine. The INIT subroutine sets all of our counters and subscripts to zero at this time and initializes the matrices which will be needed to model the artificial boundary.

2. Subroutine DOMAIN

The DOMAIN subroutine calculates the pressure at all nodes in the interior of the fluid medium at a subsequent time level in terms of the preceding pressure values. The subroutine calculates the pressure at the next time step by use of the differenced wave Equation 2.70.

The equation above must be modified to model what is happening at the side boundaries of the fluid domain. As was stated earlier, our model is looking at a small section of an "infinite" fluid plate interface. This allows us to use the pressure in the next section to model what is happening in the section in which we are interested. This

is necessary since otherwise we would have to use "undefined" points in our differenced wave equation at the edges of the fluid domain.

3. Subroutine FLPL

The FLPL subroutine models the fluid-plate interface along the top of the reinforced thin plate. Again, with finite differencing and the assumption that we are only looking at a small section of an infinite interface, we are able to model the effects of the pressure wave along the entire fluid-plate interface. This subroutine is also where we introduce our driving function into the computer program. This function is what starts things moving towards a steady state. Although this is not the most complicated subroutine in the entire program it is the one which has proved to be the hardest to make work as predicted.

4. Subroutine ARTF

The ARTF subroutine is the most complicated of any subroutine in the computer program. It models the artificial boundary of the fluid medium at infinity. Again, this is accomplished with finite differencing and matrix operations.

5. Subroutine ALPHA

The ALPHA subroutine calculates the magnitude of the radiating modes of the pressure wave. This data is gathered in the results section of this study.

6. Subroutines SHIF and FINN

The SHIF subroutine moves the entire problem to the next time level. This is done after the other subroutines have completed all the required calculations needed

to satisfy our domain and all of our boundary conditions. After exiting the SHIF subroutine the entire program is run again with the exception of the INIT subroutine.

The FINN subroutine is a check to make sure that we have reached a steady state situation before we take any measurements of the plate displacement or of any fluid disturbances. Figure 5 is a graphical representation of the displacement convergence factor approaching zero. This was arrived at with a assigned frequency of 3500 Hz and L = 1.0.

7. Subroutine CLOSE

The CLOSE subroutine is used to send the data generated in other parts of the main program to the plotting devices.

C. PROGRAM VERIFICATION

Verification of the program was accomplished by the following method. We solved the waveguide problem (section 2E) for a known value of g(x). We let $g(x) = \sin \pi x$ and $e^{i\pi x}$. This was accomplished by replacing the FLPT subroutine with a subroutine named TEST. As before the program was run until a steady state solution was reached. These results were checked against analytical solutions and found to compare very well. For $g(x) = \sin \pi x$ we graphically reproduced the expected results. For $g(x) = e^{i\pi x}$ we calculated the coefficients α_n and found $\alpha_1 \sim 1$ and all other α_n's ~ 0. The small error found in the calculation can be attributed to truncation errors in the finite difference approximations. These results match with analytical calculations.

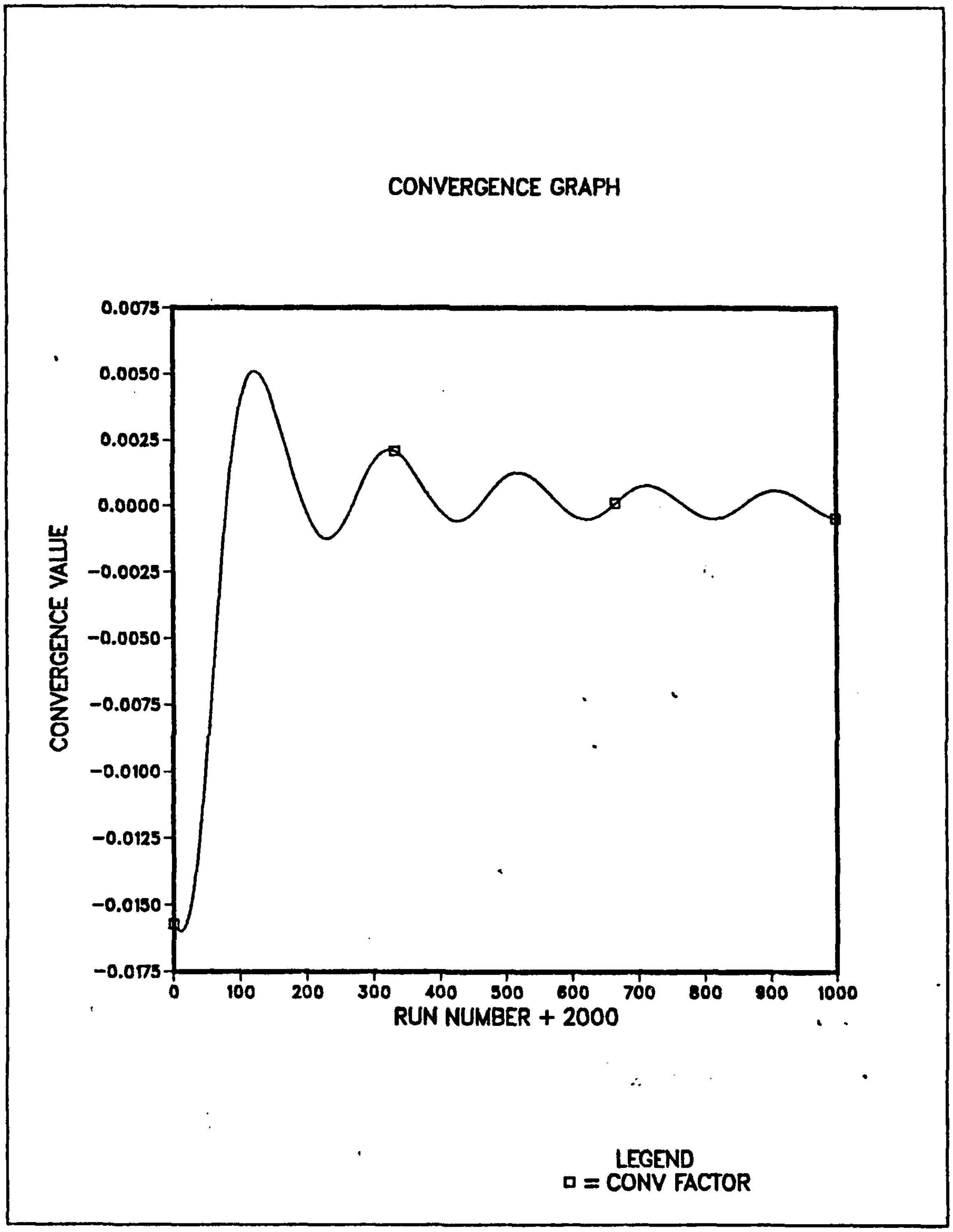


Figure 5. Convergence Graph

IV RESULTS AND CONCLUSIONS

A. INITIAL CONDITIONS

To properly apply the computer program generated, inputted data, or conditions, must be within the tolerances of the main equations used. This is demonstrated by the use of .01 meters as the thickness of the thin plate and a frequency range from 500 Hz to 4000 Hz. Other inputted data, including the densities of steel and water were taken directly from Kinsler, Frey, Coppens and Sanders [Ref. 1:pp. 461-463].

The choice of the Δx and Δy terms is based on the need to have a significant number of data points in the waveguide domain that will give an accurate picture of what is happening. This is also true for the time step chosen, we need to make sure that the Δt was small enough to keep the problem from blowing up. A Courant-Lewy-Friedrichs stability condition must be satisfied for the domain which limits the size of Δt. Additionally as was mentioned before, the computer program was run for the same amount of time for each trial, this time period is long enough to ensure that a steady state solution was being achieved.

B. PROGRAM OUTPUT DATA

The data obtained from the computer program demonstrates the following major points. When the frequency is increased you get more propagating modes from each particular plate length. When L is changed by a specific amount you do see a difference in the magnitude and shape of the deformation graphs.

Our first table shows the calculated magnitudes of the propagating modes of the pressure wave at an assigned frequency of 750 Hz at different beam spacings.

Table I. MAGNITUDE OF RADIATING MODES.

For a frequency of 750 Hz	
with L = 0.8	α_0 = .467808127
with L = 1.0	α_0 = .484098911
	α_1 = .307484090
with L = 1.2	α_0 = .232444227
	α_1 = .367214561

In addition to the data in Table I, we have in Figures 6 through 8 a graphical representation of the scalded pressure amplitude. The pressure was measured at the artificial boundary , y_B at a frequency of 750 Hz. The horizontal axis is of a length of 2L for each of the graphs starting at $\bar{x}$ = -1 and going to $\bar{x}$ = 1. The numbering on the horizontal axis represents each of the 41 pressure nodes used in the finite difference code.

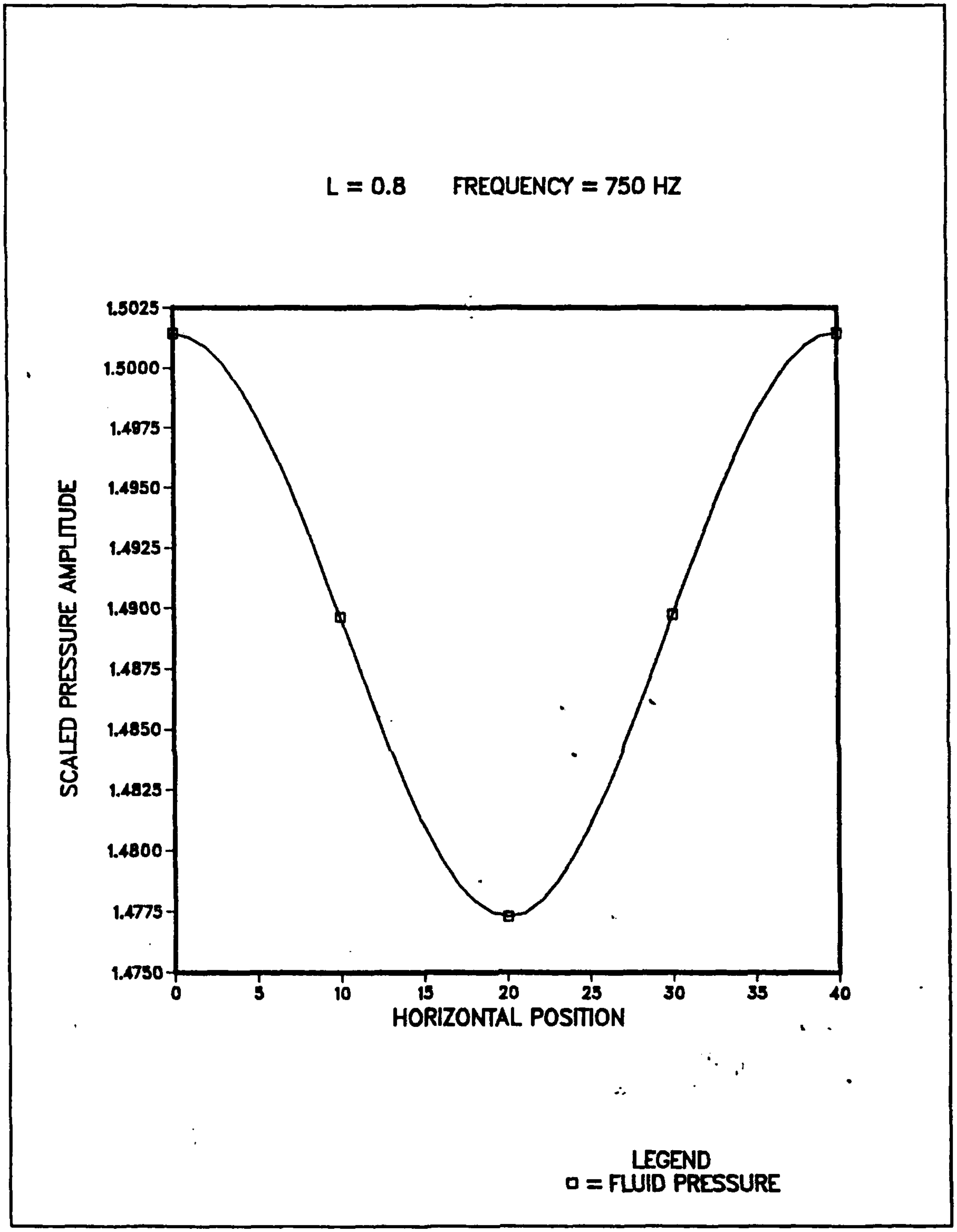


Figure 6. Sample Pressure Graph

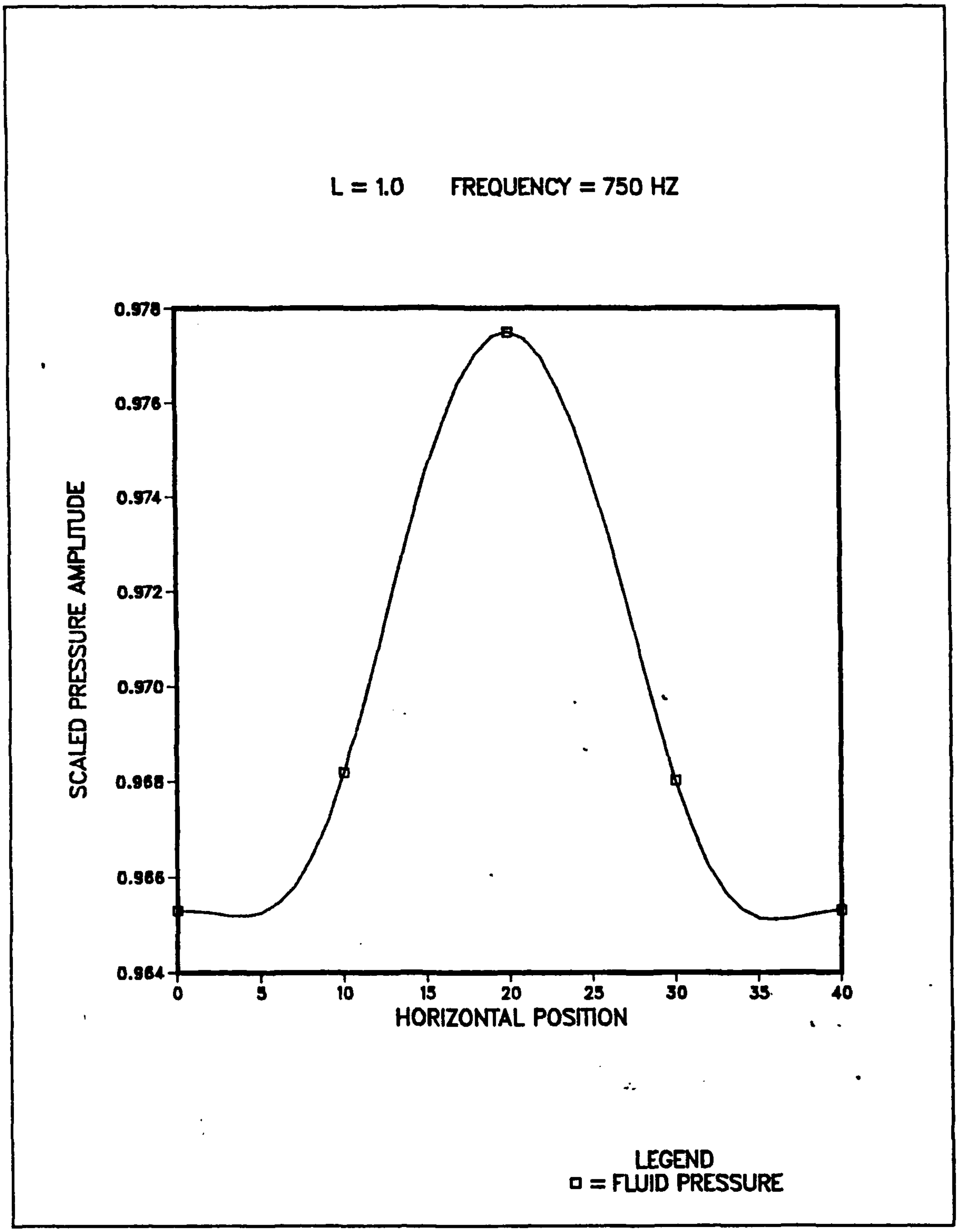


Figure 7. Sample Pressure Graph

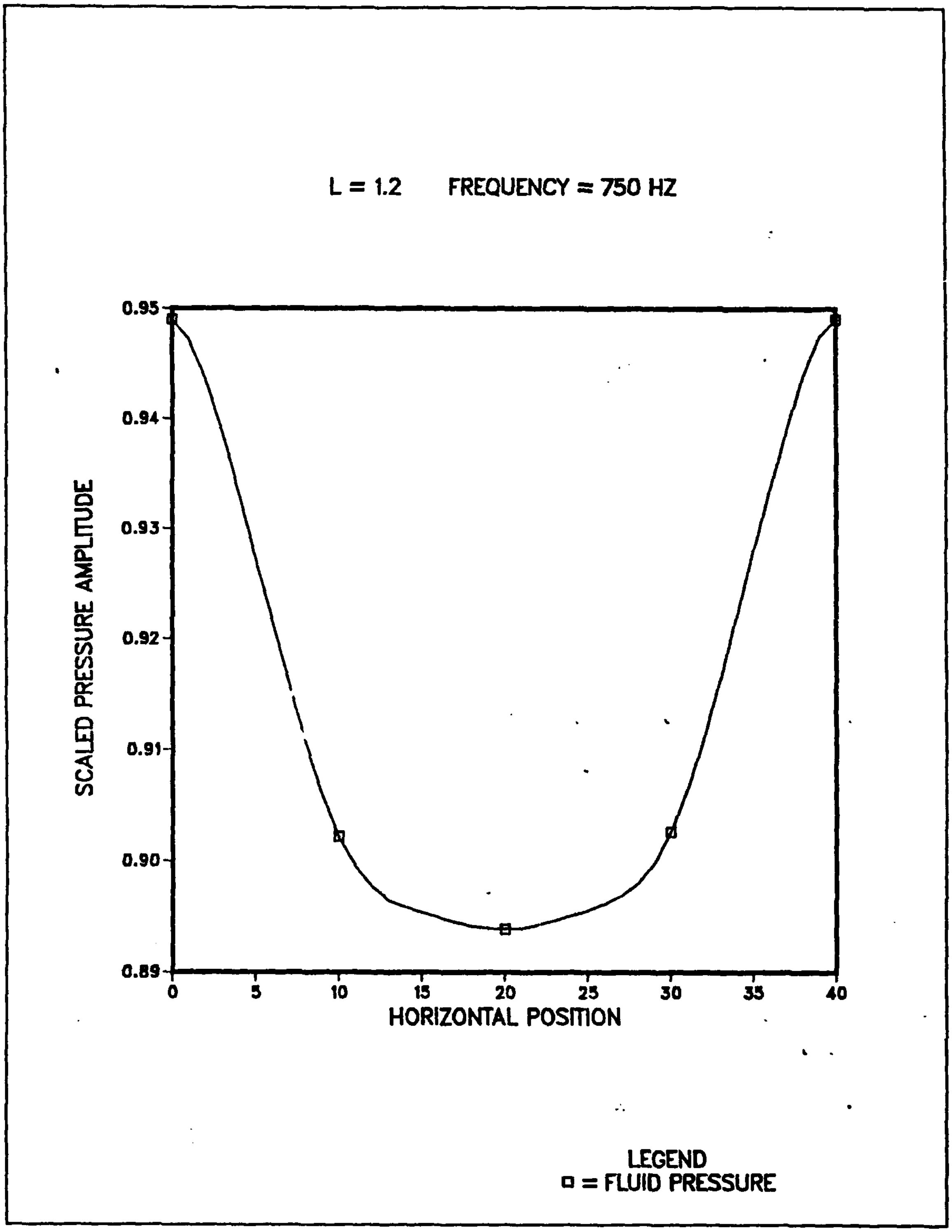


Figure 8. Sample Pressure Graph

Table II is a list of the magnitudes of the α_n's for program runs with a frequency of 3500 Hz.

Table II. MAGNITUDE OF RADIATING MODES.

For a frequency of 3500 Hz

with L = 0.8	α_0 = .451181054
	α_1 = .323620617
	α_2 = .088650584
with L = 1.0	α_0 = .365310729
	α_1 = .260265648
	α_2 = .062102031
	α_3 = .060141488
	α_4 = .076183497
with L = 1.2	α_0 = .252027571
	α_1 = .165709317
	α_2 = .047427550
	α_3 = .046469088
	α_4 = .049672558
	α_5 = .043821129
	α_6 = .067380726

From the data above we observe that at a frequency of 3500 Hz there is a substantial change in the magnitude of the propagating modes as L is increased from 0.8 to 1.2. It can be seen from other runs that the higher the frequency the larger the differences in the magnitudes of the propagating modes.

Figures 8 through 10 show the scaled pressure amplitude data obtained from the program runs with an assigned frequency of 3500 Hz. It is observed that changing of the length L on which the Δx is scaled has an effect on not only the amplitude of the pressure wave but also their shape.

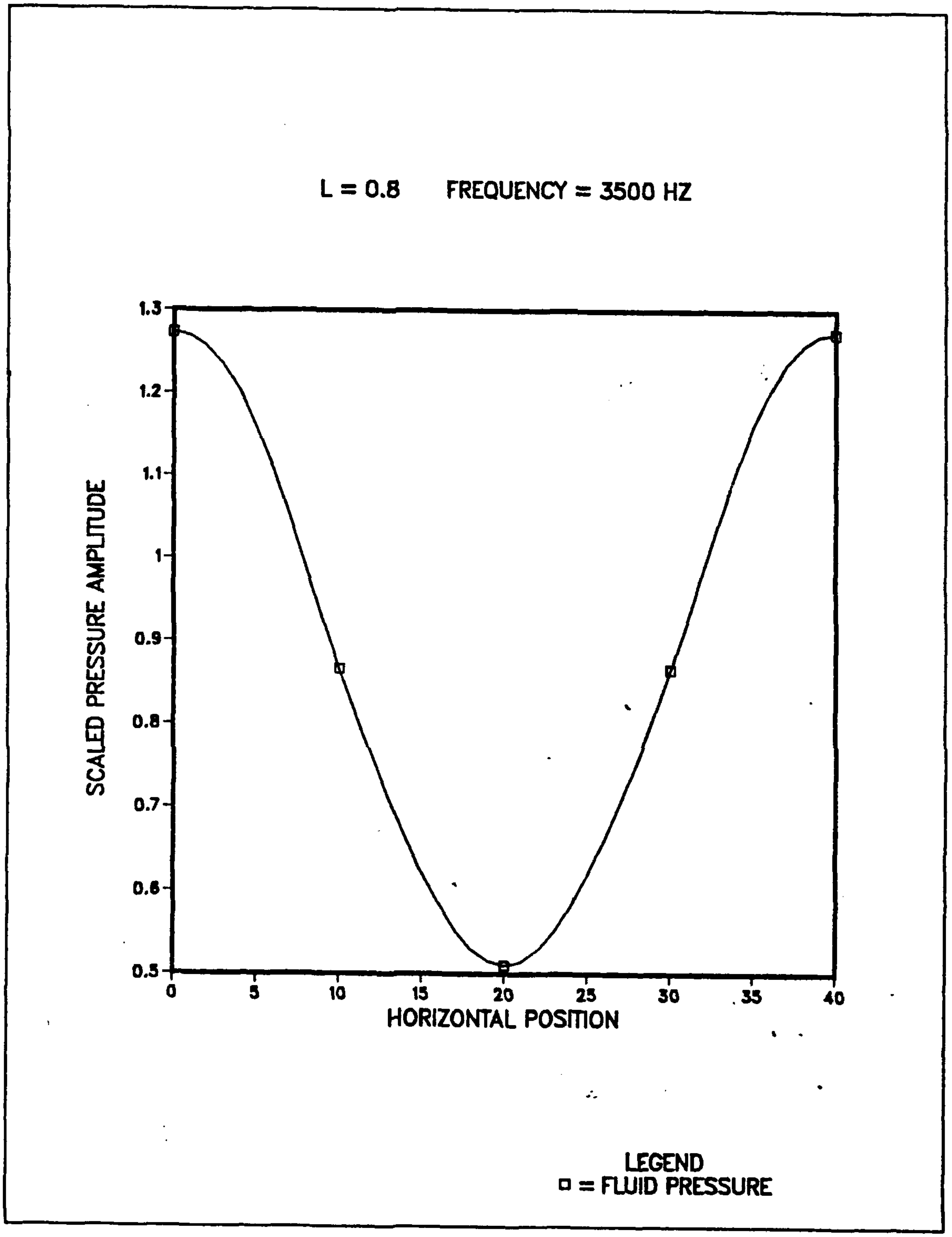


Figure 9. Sample Pressure Graph

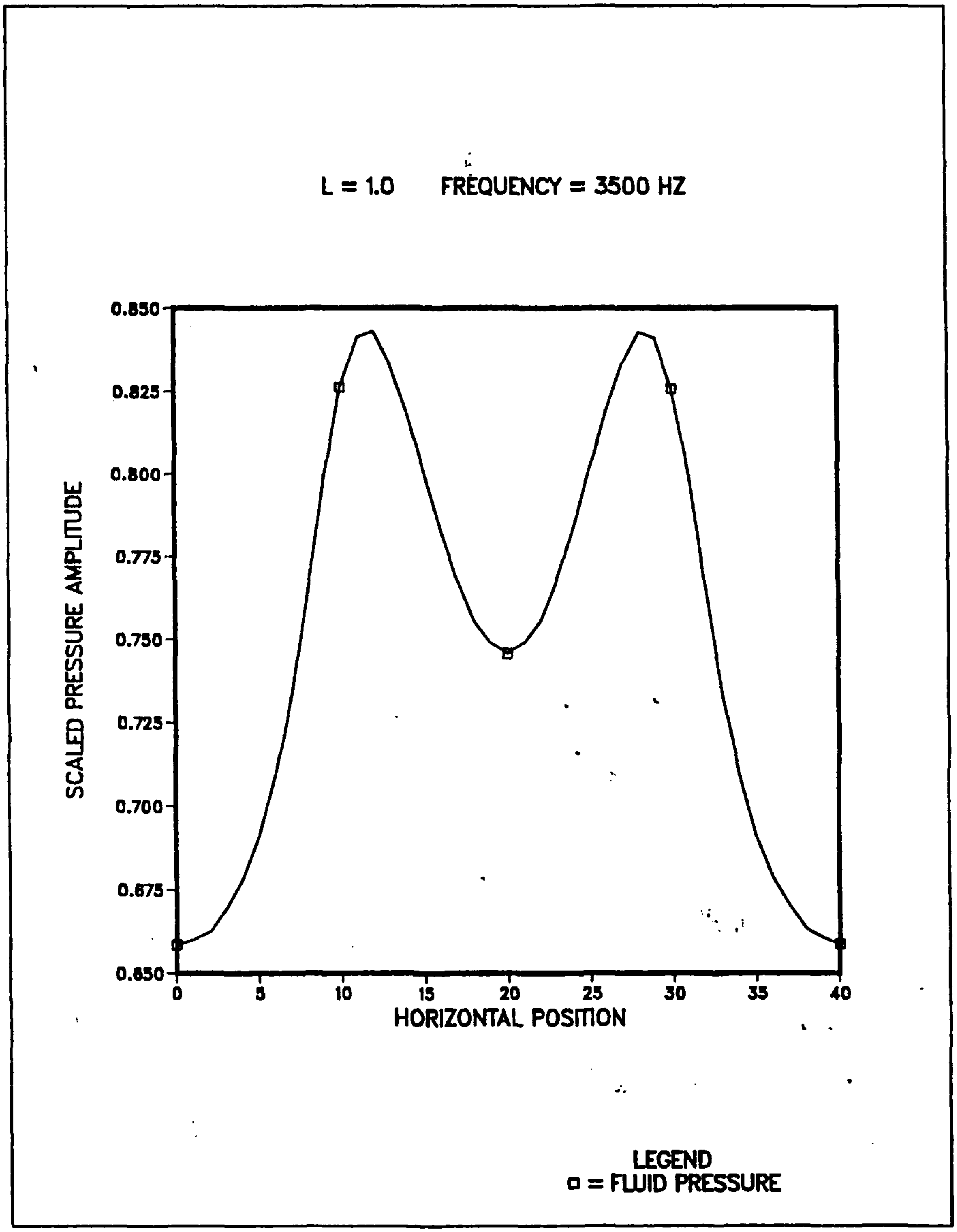


Figure 10. Sample Pressure Graph

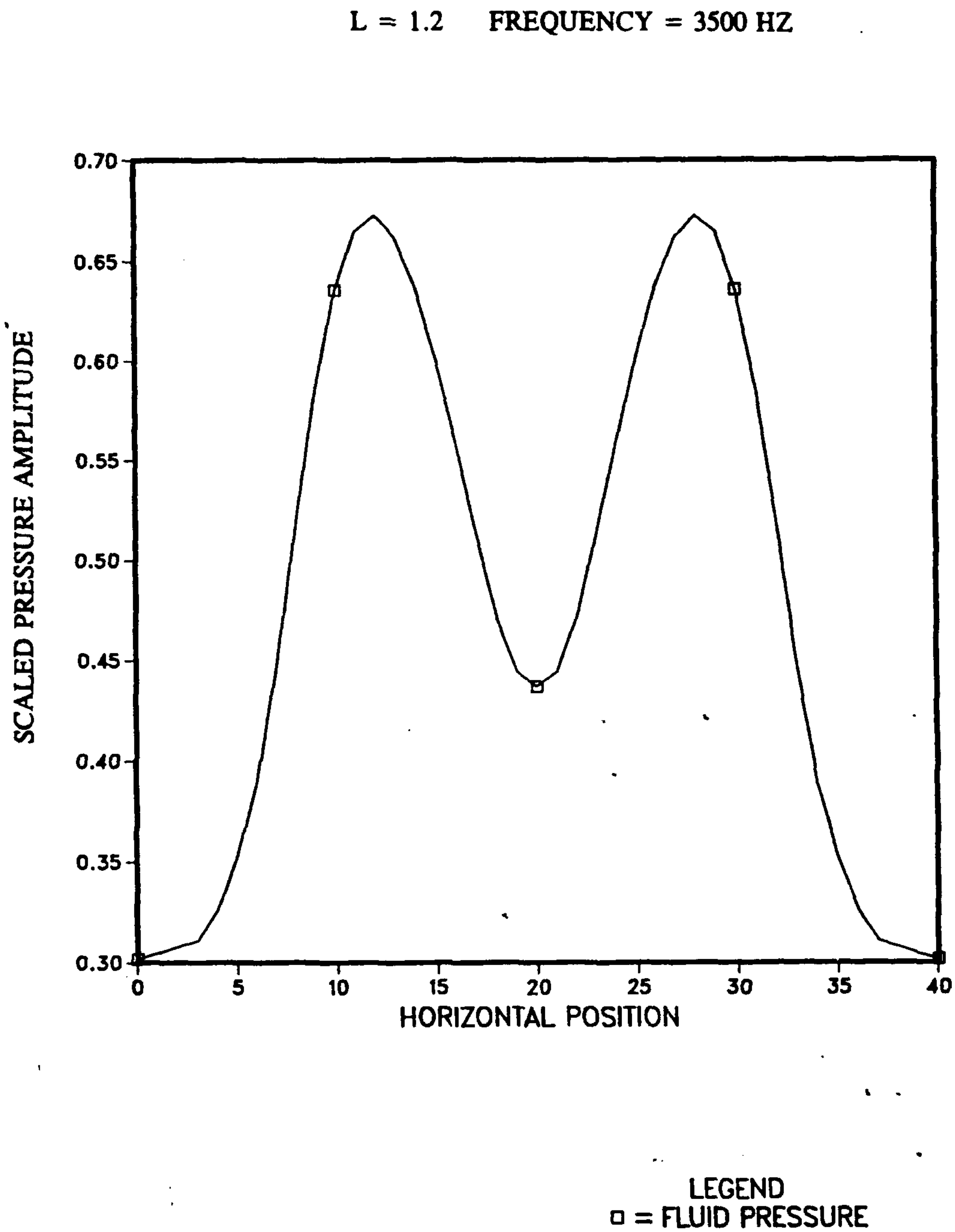


Figure 11. Sample Pressure Graph

Table III is a list smaple energy calculations taken at three frequencies inputted into the computer program. The data presented demonstrates that for theses frequencies and lengths run in the computer program that the fundamental mode contains the largest amount of energy. Also evident is that the energy contained in the fundamental mode starts becomes more constant as the frequency is increased for different lengths of L. This data is in agreement with the theory of sound propagation.

Table III ENERGY CALCULATIONS

		Percent of Energy in Propagating Mode
For a frequency of 750 Hz		
with L = 0.8	E_0 =	100%
with L = 1.0	E_0 =	99%
	E_1 =	1%
with L = 1.2	E_0 =	77%
	E_1 =	23%
For a frequency of 1750 Hz		
with L = 0.8	E_0 =	77%
	E_1 =	23%
with L = 1.0	E_0 =	68%
	E_1 =	31%
	E_2 =	1%
with L = 1.2	E_0 =	64%
	E_1 =	32%
	E_2 =	3%
	E_3 =	1%

		Percent of Energy in Propagating Mode
For a frequency of 3500 Hz		
with L = 0.8	E_0 =	66%
	E_1 =	32%
	E_2 =	2%
with L = 1.0	E_0 =	65%
	E_1 =	32%
	E_2 =	2%
	E_3 =	1%
	E_4 =	*
with L = 1.2	E_0 =	64%
	E_1 =	27%
	E_2 =	2%
	E_3 =	2%
	E_4 =	2%
	E_5 =	1%
	E_6 =	2%

* denotes much less then 1%

Table IV shows the magnitude of the primary propagating mode for each of the different frequencies and lengths inputted into the computer program.

Table IV MAGNITUDES OF PRIMARY MODES

	Magnitude of α_0's		
	L = 0.8	L = 1.0	L = 1.2
Frequency			
750 Hz	.467808127	.484098911	.367214561
1000 Hz	.6182562721	.444052100	.262355387
1250 Hz	.553467214	.457091272	.343211770
1500 Hz	.498957574	.415586054	.320898831
1750 Hz	.404151559	.431289613	.338643909
2000 Hz	.486700475	.433078885	.316784620
2250 Hz	.394520938	.400619507	.252146780
2500 Hz	.385663450	.392564356	.267684340
2750 Hz	.385663450	.392564354	.267684340
3000 Hz	.420025572	.353338659	.234915197
3250 Hz	.369634628	.226448834	.194508076
3500 Hz	.451181054	.365310729	.252027571
3750 Hz	.446322470	.330691218	.226533830

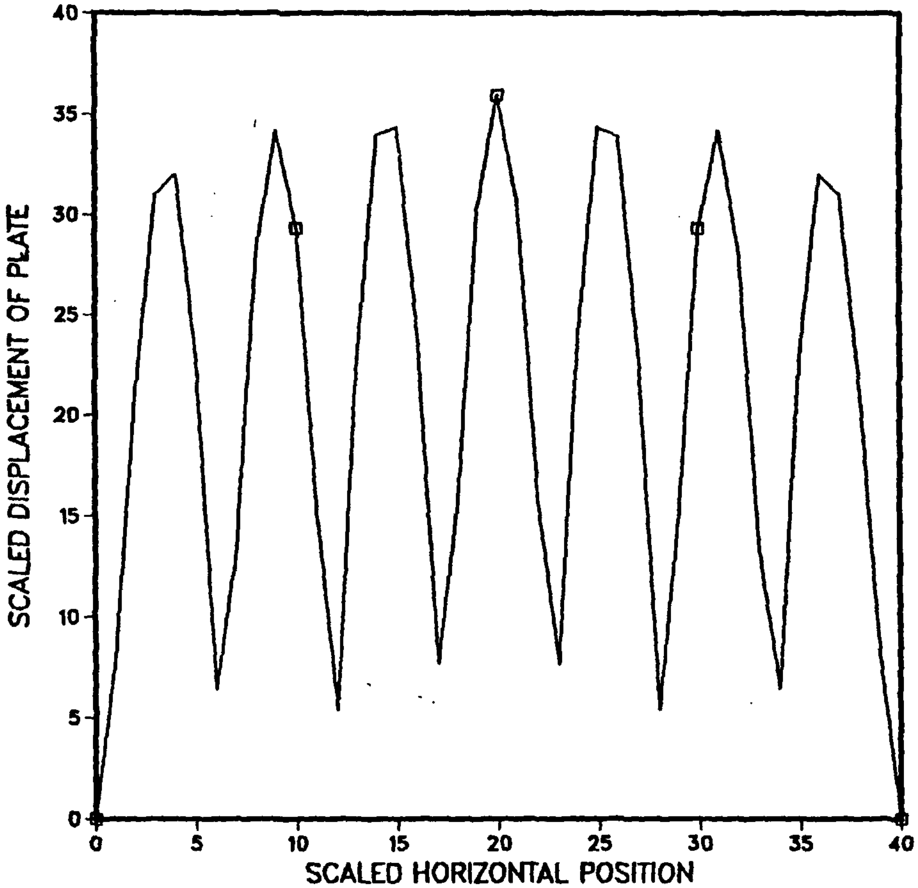
The data obtained from the computer program and represented in Table III shows that there is always a difference in the magnitude of the fundamental propagating mode for changes in L at all frequencies comprised within the computer runs. The difference in the magnitude of the primary propagating mode between the runs with a length "L" of 1.2 and the runs with a length L of 1 are greater then the difference between the runs with lengths of 0.8 and 1.0. In the frequency range covered when L is increased from 1.0 to 1.2 there is always a drop in the magnitude of the fundamental propagating mode. This is not always true when L is decreased to 0.8 from 1.0. Although the magnitude of these increases are smaller then the magnitude of the decreases this demonstrates that by increasing the spacing of the reinforcing beams will not always result in a decrease in the amplitude of the fundamental propagating mode in this frequency range. From this result we can conclude that additional factors play an important role in determining the magnitude of the propagating modes.

Because the model is allowed to run until a steady state solution is reached there is no appreciable difference in the absolute pressure measured at the artificial boundary for any of the frequencies observed. However, it is evident that as the length of the plate "L" is increased there is a slight drop in the pressure measurement. This is true for all frequencies. Figure 12 is a sample plate deformation graph for L = 1.0 and a frequency of 1000 Hz.

In conclusion, the computer program produces results as expected for the different frequencies of the plane wave and lengths of the thin plate. The superpositioning of the reflected plane wave has showed that as L is increased there is a larger number of

propagating modes accompanied by a change in the amplitude of each mode. Additional results could be explored for different angles of deflection as well as different plate thicknesses. This will require modification of the computer program as it stands to gather data for additional angles and thicknesses.

SCALED LENGTH OF 1.0



LEGEND
□ = DISP AT 1000 HZ

Figure 12. Deformation Graph

APPENDIX: COMPUTER PROGRAM

```
FILE: 28OCT    FORTRAN  A1

C*****************************************************
C
C     . THIS PROGRAM SIMULATES THE DISTRUBANCE IN A FLUID
C       AFTER A SOUND WAVE HAS HIT A PLATE.
C
C       P = PRESSURE (EXCESS)      D = FLEXURAL RIGIDITY
C       X = HORIZONTIAL POSITION   Y = VERTICAL POSITION
C       W = DEFORMATION           KF = FLUID WAVE NUMBER
C     SPD = WAVE SPEED            KP = PLATE WAVE NUMBER
C    FREQ = FREQUENCY          OMEGA = RATIO OF (KF/KP)**2
C
C       COUNTERS:  I  J  N
C
C          THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325  )
      PARAMETER (NDIM = 2*IX+1)
C           THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16    P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX       LB, FACR
      COMPLEX       A( -IX: IX, -IX: IX)
      COMPLEX       AA( NDIM, NDIM)
      COMPLEX       BB( NDIM, NDIM)
      COMPLEX       C( -IX: IX)
      COMPLEX       Z( NDIM )
      COMPLEX*16    F1
      COMMON/SS/P, W
      COMMON/TT/KP,KF,INFA, THETAI,L, X, PI,
     1          EISP, OMEGA, T
      COMMON/CC/LB, FACR
      COMMON/EE/WGTWO, WGTHE, WGTOT, CON
      COMMON/PT/PGTWO, PGTHE, PGTOT
      COMMON/HH/MINN, MAXX
      COMMON/II/AA, BB
      COMMON/KK/IPVT
      COMMON/MM/NN
      COMMON/OO/C
      COMMON/FF/A
      COMMON/UP/AL
      COMMON/RN/RUNS
      COMMON/FEL/U,V,TP
      COMMON/SN/FREQ, SPD
      COMMON/GF/WGGRF,PGGRF
C
      INTEGER  RUNS, IPVT(NDIM), SAM
      PARAMETER (SAM = 1000)
      REAL      KF, KP, INFA, EISP, OMEGA, L,PI
      REAL*8    WGTWO,WGTHE,WGTOT,CON
      REAL*8    PGTWO,PGTHE,PGTOT
      REAL      WGGRF( SAM ), PGGRF( SAM )
      REAL      THETAI, FREQ, SPD
      REAL      X, T
      PI = 3.141592654
      CON  = .015D0
C
```

```
C
C
C
C
C
        WGTWO = 0.0D0
        WGTHE = 0.0D0
        PGTWO = 0.0D0
        PGTHE = 0.0D0
C
C
      CALL XUFLOW (0)
      CALL INIT
         RUNS = 0
   20 CALL FLPL
      CALL DOMAIN
         RUNS = RUNS + 1


      CALL ARTF
      IF (RUNS .GT. 1999) THEN
      GOTO 60
      ELSE
      GOTO 61
      ENDIF
   60 CALL FINN
   61    T = T + DELTAT
      CALL SHIF
      IF (RUNS .EQ. 3000) THEN
      GOTO 80
      ELSE
      GOTO 20
      ENDIF
   80 CALL ALPHA
      CALL CLOSE
C
      WRITE(2,767)THETAI,KF
  767 FORMAT(1X,'THETAI = ',F12.6,3X,'  KF = ',F12.6)
      WRITE(2,768)OMEGA,EISP
  768 FORMAT(1X,' OMEGA = ',F12.6,3X,'EISP = ',F12.7)
      WRITE(2,196)DELTAT, KP
  196 FORMAT(1X,'DELTAT = ',F12.6,3X,'  KP = ',F12.6)
      WRITE(2,487)H,L
  487 FORMAT(1X,'      H = ',F12.6,3X,'   L = ',F6.3)
      WRITE(2,735)FREQ,SPD
  735 FORMAT(1X,'FREQ = ',F12.6,3X,'SOUND SPEED = ',F12.6)
      WRITE(2,629)PI
  629 FORMAT(1X,'  PI = ',F12.7)
      WRITE(2,919)
  919 FORMAT(1X,'FACR = ')
      WRITE(2,*) FACR
C
```

```
C
C
C
C
C
        WRITE(2,222)
  222   FORMAT(1X,'FINAL DATA',//,1X,'       X     ',
     1  '     W     ','     P     ')
        DO 333 I = -IX, IX, 1
        XX = I*H
  333   WRITE(2,666)XX,CDABS(W(I,2)),CDABS(P(I,IY,3))
  666   FORMAT(1X,5F10.5)
        WRITE(2,223)RUNS
  223   FORMAT(1X,'RUNS = ',I7)
        WRITE(2,678)
  678   FORMAT(1X,'BELOW IS THE FINAL WGTOT, AND PGTOT')
        WRITE(2,*)WGTOT
        WRITE(2,*)PGTOT
      STOP
      END
C
      SUBROUTINE INIT
C
C        THIS SETS UP THE INITIAL CONDITIONS OF THE
C        PROBLEM INCLUDING THE DOMAIN AND THE
C        ARTIFICIAL BOUNDARY
C
C        THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
      PARAMETER (NDIM = 2*IX+1)
C         THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16   P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX      LB, FACR
      COMPLEX      A( -IX:IX, -IX: IX)
      COMPLEX      AA( NDIM   , NDIM   ), BB( NDIM   , NDIM   )
      COMPLEX      Z(  NDIM  )
      COMMON/SS/P, W
      COMMON/TT/KP,KF,INFA, THETAI,L, X, PI,
```

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```
     1          EISP, OMEGA, T
      COMMON/CC/LB, FACR
      COMMON/FF/A
      COMMON/GG/GAMMA, BETTA
      COMMON/HH/MINN, MAXX
      COMMON/II/AA, BB
      COMMON/KK/IPVT
      COMMON/FEL/U,V,TP
      COMMON/SN/FREQ, SPD
C
      REAL      THETAI, INFA, KF, EISP, OMEGA, TIN, L, KP
      REAL      ART, K, XI, XIL, RCOND
      REAL      GAMMA(-40: 40), BETTA (-40:40), PI
      REAL      SPD, FREQ
      REAL      X, T
      INTEGER I, J, N, LJ, NN, U, V, TP
      INTEGER  NMODMIN, NMODMAX, MINN, MAXX
      INTEGER  IPVT(NDIM)
```

```
C
      TP = 41
      LB = CMPLX(0.0, 1.0)
      FACR = CMPLX(1.0, 0.0)
      SPD = 1500.0D0
      FREQ = 3750.0D0
      N = 0
      I = 0
      J = 0
      NN = 1
      L =  1.2D0
      THETAI =  PI/2.0D0
      T = DELTAT
       NMODMIN = 0
       NMODMAX = 0
       KF = (2D0*PI*FREQ*L)/SPD
C      GT = ((195000000000.0*(.05**3))/12(1-(.28**2))
C      KP = ((((FREQ*2*PI)**2)*7700*H)/FR)**.25
       KP = 40.7534975147D0
       OMEGA = (KF/KP)**2
      INFA = (DELTAT**2)/((KF**2)*(L**2)*(H**2))
      DO 12 I = -IX,IX,1
        DO 11 J = 0,IY,1
           P(I,J,1) = CMPLX(0.0, 0.0)
           P(I,J,2) = CMPLX(0.0, 0.0)
           P(I,J,3) = CMPLX(0.0, 0.0)
   11 CONTINUE
   12 CONTINUE
C
      DO 21 I = -IX, IX, 1
         W(I,1) = CMPLX(0.0, 0.0)
         W(I,2) = CMPLX(0.0, 0.0)
C        W(I,3) = CMPLX(0.0, 0.0)
   21 CONTINUE
C
C         THIS SETS UP THE ARTIFICAL BOUNDRAY
C
      K = KF*L
      TIN = -K*COS(THETAI)
      DO 15 N = -40, 40, 1
         GAMMA(N) = TIN + PI*N
           IF ( ABS(GAMMA(N)) .LE. K) THEN
             NMODMIN = MIN(N,  NMODMIN)
             NMODMAX = MAX(N,  NMODMAX)
           ENDIF
   15 CONTINUE
                MINN = NMODMIN
                MAXX = NMODMAX
C

     DO 18 N = MINN, MAXX, 1
        BETTA(N) =   SQRT((K**2) - (GAMMA(N)**2))
  18 CONTINUE
C
     DO 78 I = -IX, IX, 1
       DO 79 LJ =  -IX, IX, 1
            A(I,LJ) = CMPLX(0.0,0.0)
  79   CONTINUE
  78 CONTINUE
C
     ART = (2*(H**2))/(4*DELTAT)
     DO 300  I = -IX, IX, 1
        DO 301  LJ = -IX, IX, 1
          XI = I*H
          XIL = LJ*H
            IF (ABS(LJ) .EQ. IX) THEN
C
```

```
C
              DEL = .5
              ELSE
              DEL = 1
           ENDIF
        DO 302 N = MINN, MAXX, 1
        A(I,LJ) = A(I,LJ) + ART*DEL
     1                   *BETTA(N)*CEXP(LB*GAMMA(N)*(XI-XIL))
  302 CONTINUE
  301 CONTINUE
  300 CONTINUE
C
         DO 420 U = 1, NDIM, 1
         DO 421 V = 1, NDIM, 1
                    I = -IX+U-1
                   LJ = -IX+V-1
            AA(U,V) =  INFA*(A(I,LJ))
            BB(U,V) =  INFA*(A(I,LJ))
      IF (I .EQ. LJ) THEN
            AA(U,V) = AA(U,V) + 1.0
            BB(U,V) = BB(U,V) - 1.0
      ENDIF
  421 CONTINUE
  420 CONTINUE
        CALL CGECO (AA,NDIM,NDIM, IPVT, RCOND,Z)
      RETURN
      END
C
      SUBROUTINE FLPL
C
C        THIS SUBROUTINE DEFINES THE FLUID-PLATE INTERFACE
C        THIS IS WHERE Y=0 ALONG THE TOP OF THE PLATE.
C
C        THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
      PARAMETER (NDIM = 2*IX+1)
C         THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16    P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX       LB, FACR, PTEMP
      COMPLEX*16    FI
      COMMON/SS/P, W
      COMMON/TT/KP,KF,INFA, THETAI,L, λ, PI,
     1             EISP, OMEGA, T
      COMMON/CC/LB, FACR
      COMMON/RN/RUNS
C
      REAL     KP, L, OMEGA, EISP, KF, INFA, PI, THETAI
      REAL     X, T
      REAL*4   SQ,SW,BW1,BW2,BW3,BW4
      INTEGER I, J, RUNS

      EISP = 0.13101864566D0
C
        J = 0
        SQ  = (DELTAT/(H*KF))**2
        SW  = (DELTAT/(H*KP)**2)**2
        BW1 = 2-6*SW
        BW2 = 4*SW
        BW3 = EISP*DELTAT**2*KF/OMEGA
        BW4 = -SW
        A = FLOAT(IX)/FLOAT(2*IX)
C
```

```
C
C       THIS LOOP COMPUTES THE INTERIOR NODES
C
        DO 10 I = -IX + 2, IX -2, 1
           X = I*H
      W(I,3) = BW1*W(I,2) -W(I,1) + BW2*(W(I+1,2)+W(I-1,2))
     1          +BW4*(W(I+2,2)+W(I-2,2))
     2          +BW3*(P(I,0,2)+ F1(X,T))
      PTEMP      = -(2*H/DELTAT**2)*(W(I,3)-2*W(I,2)+W(I,1)) +P(I,1,2)
      P(I,J,3) = INFA*(P(I+1,J,2)+P(I-1,J,2)-4*P(I,J,2)+P(I,J+1,2)+
     1             PTEMP      )+2*P(I,J,2) -P(I,J,1)
   10 CONTINUE
C
C       THESE LOOPS COMPUTE THE NODES AT -19 AND 19
C
      I = -IX+1
      X = (-IX+1)*H
      W(I,3) = BW1*W(I,2)-W(I,1) +BW2*(W(I+1,2)+W(I-1,2))
     1          +BW4*(W(I+2,2)+W(I,2))
     2          +BW3*(P(I,0,2)+ F1(X,T))
      PTEMP      = -(2*H/DELTAT**2)*(W(I,3)-2*W(I,2)+W(I,1)) +P(I,1,2)
      P(I,J,3) = INFA*(P(I+1,J,2)+P(I-1,J,2)-4*P(I,J,2)+P(I,J+1,2)+
     1             PTEMP      )+2*P(I,J,2) -P(I,J,1)
C
      I = IX -1
      X = (IX-1)*H
      W(I,3) = BW1*W(I,2) - W(I,1) + BW2*(W(I+1,2)+W(I-1,2))
     1          +BW4*(W(I,2)+W(I-2,2))
     2          +BW3*(P(I,0,2)+ F1(X,T))
      PTEMP      = -(2*H/DELTAT**2)*(W(I,3)-2*W(I,2)+W(I,1)) +P(I,1,2)
      P(I,J,3) = INFA*(P(I+1,J,2)+P(I-1,J,2)-4*P(I,J,2)+P(I,J+1,2)+
     1             PTEMP      )+2*P(I,J,2) -P(I,J,1)
C
C         THESE ARE THE END POINTS
C
      FACR=CEXP(2*LB*L*KF*COS(THETAI))
      I = -IX
      J = 0
      W(I,3) = CMPLX(0.0,0.0)
      PTEMP  = P(I,1,2)
      P(I,J,3) = INFA*(P(I+1,J,2)+FACR*P(-I-1,J,2)-4*P(I,J,2
     1             +P(I,J+1,2)+PTEMP      )+2*P(I,J,2) -P(I,J,1)
      FACR=CEXP(-2*LB*L*KF*COS(THETAI))
      I = IX
      J = 0
      W(I,3) = CMPLX(0.0,0.0)
      PTEMP  = P(I,1,2)
      P(I,J,3) = INFA*(FACR*P(-I+1,J,2)+P(I-1,J,2)-4*P(I,J,2)
     1             +P(I,J+1,2)+PTEMP      )+2*P(I,J,2) -P(I,J,1)
      RETURN
      END
C
```

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```
      SUBROUTINE DOMAIN
C
C        THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
      PARAMETER (NDIM = 2*IX+1)
C         THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16   P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX      LB, FACR
      COMMON/SS/P, W
      COMMON/TT/KP, KF, INFA, THETAI, L, X, PI,
     1          EISP, OMEGA ,T
      COMMON/CC/LB, FACR
C
      REAL    KP, KF, INFA, THETAI, L, PI, EISP, OMEGA
      REAL    X, T
      INTEGER I,J
C
      DO 22 I = 1-IX, IX-1,1
         DO 21 J = 1, IY-1, 1
         P(I,J,3) = INFA*(P(I+1,J,2)+P(I-1,J,2)-
     1             4*P(I,J,2)+P(I,J+1,2)+P(I,J-1,2))+
     2             2*P(I,J,2)-P(I,J,1)
   21    CONTINUE
   22 CONTINUE
      LB = CMPLX (0.0, 1.0)
      LA = L*KF*COS(THETAI)
      FACR = CEXP((2)*(LB*LA))
      I = -IX
      DO 41 J = 1, IY-1   , 1
      P(I,J,3) = INFA*(P(I+1,J,2)+ FACR*P(-I-1,J,2)
     1           -4*P(I,J,2)+P(I,J+1,2)+P(I,J-1,2))+
     2           2*P(I,J,2)-P(I,J,1)
   41 CONTINUE
C
      LB = CMPLX (0.0, 1.0)
      RA = L*KF*COS(THETAI)
      FACR = CEXP((-2)*(LB*RA))
      I = IX
      DO 31 J = 1, IY-1,1
      P(I,J,3) = INFA*(FACR*P(-I+1,J,2)+ P(I-1,J,2)
     1           -4*P(I,J,2)+P(I,J+1,2)+P(I,J-1,2))+
     2           2*P(I,J,2)-P(I,J,1)
   31 CONTINUE
      RETURN
      END
C
C
```

```
C
      SUBROUTINE ARTF
C
C        THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
      PARAMETER (NDIM = 2*IX+1)
C         THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16   P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX      LB, FACR
      COMPLEX      A( -IX: IX, -IX: IX)
      COMPLEX      AA( NDIM   , NDIM   ), BB( NDIM   , NDIM   )
      COMPLEX      B(NDIM)
      COMPLEX      C( -IX: IX)
      COMPLEX      Z(  NDIM  )
      COMMON/SS/P, W
      COMMON/TT/KP,KF,INFA, THETAI,L,X, PI,
     1          EISP, OMEGA ,T
      COMMON/CC/LB, FACR
      COMMON/FF/A
      COMMON/II/AA, BB
C
      SUBROUTINE SHIF
C
C         THIS MOVES THE PROGRAM TO THE NEXT TIME LEVEL
C
C        THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
      PARAMETER (NDIM = 2*IX+1)
C         THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16   P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX      LB, FACR
      COMMON/SS/P, W
      COMMON/TT/KP,KF,INFA, THETAI, L, X, PI,
     1          EISP, OMEGA, T
      COMMON/CC/LB, FACR
C
      REAL    KP, L, OMEGA, EISP, KF, INFA, PI, THETAI
      REAL    X, T
      INTEGER  I, J
C
```

```
      COMMON/FEL/U,V,TP
      COMMON/KK/IPVT
      COMMON/OO/C
C
      REAL     KP, L, OMEGA, EISP, KF, INFA, PI, THETAI
      REAL     X, T
      INTEGER   IPVT(NDIM), JOB ,U,V, TP
      TP = 41
      JOB = 0
      LB = CMPLX (0.0, 1.0)
      LA = L*KF*COS(THETAI)
      RA = L*KF*COS(THETAI)
C
      DO 42 I = -IX+1, IX-1, 1
      C(I) = INFA*(P(I+1,IY,2)+P(I-1,IY,2)+2*P(I,IY-1,2))
     1       + (2 - (4*INFA))*P(I,IY,2)
   42 CONTINUE
      I = -IX
      FACR = CEXP((2)*(LB*LA))
      C(I) = INFA*(P(I+1,IY,2)+FACR*P(-I-1,IY,2)+2*P(I,IY-1,2))
     1       + (2 - (4*INFA))*P(I,IY,2)
      I = IX
      FACR = CEXP((-2)*(LB*RA))
      C(I) = INFA*(FACR*P(I-1,IY,2)+P(-I+1,IY,2)+2*P(I,IY-1,2))
     1       + (2 - (4*INFA))*P(I,IY,2)
C
      DO 10 I =   1, NDIM, 1
         B(I) = INFA*(CMPLX(0.0, 0.0))
   10 CONTINUE
C
C
      DO 20 I = 1,NDIM,1
         DO 30 LJ = 1, NDIM, 1
            B(I) = B(I) + BB(I,LJ)*P(-IX+LJ-1,IY,1)
   30 CONTINUE
   20 CONTINUE
C
        DO 150 I =   1, NDIM, 1
           B(I) = B(I) + C(-IX+I-1)
  150 CONTINUE
      CALL CGESL(AA,NDIM,NDIM,IPVT,B,JOB)
      DO 46 I = -IX, IX, 1
            P(I,IY,3) = B(IX+1+I)
   46 CONTINUE
      RETURN
      END
C
```

```
      DO 19 I = -IX, IX, 1
        DO 22 J = 0, IY, 1
        P(I,J,1) =  P(I,J,2)
   22   CONTINUE
   19 CONTINUE
      DO 20 I = -IX, IX, 1
        DO 23 J = 0, IY, 1
        P(I,J,2) =  P(I,J,3)
   23   CONTINUE
   20 CONTINUE
C
      DO 29 I = -IX, IX, 1
        W(I,1) =  W(I,2)
   29 CONTINUE
      DO 40 I = -IX, IX, 1
        W(I,2) =  W(I,3)
   40 CONTINUE
      RETURN
      END
C
      FUNCTION F1(XP, TIME)
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
      COMPLEX*16   P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX*16  F1, S1
      COMMON/SS/P, W
      COMMON/RN/RUNS
      COMMON/TT/KP,KF,INFA, THETAI,L, X, PI,
     1          EISP, OMEGA, T
C
      REAL    KP, L, OMEGA, EISP, KF, INFA, PI, THETAI
      REAL    X, T
      REAL*8 Q9,Q5,S
C
C
        IF(RUNS .EQ. 1) THEN
        S = 0.0
        ELSE
      S  = TIME+XP*KF*COS(THETAI)
        ENDIF
      Q9 = 0
      Q5 = AMIN1(TIME, 18.0)
C
C
```

```
C
      Q8 = DEXP(2*Q5-6.0)
      Q7 = Q8/(Q8+.1/Q8)
      S1 = CMPLX(Q9,-S)
      F1 = Q7*CDEXP(S1)
      RETURN
      END
C
C
      SUBROUTINE ALPHA
C
C        THIS SETS UP THE INITIAL CONDITIONS OF THE
C        PROBLEM INCLUDING THE DOMAIN AND THE
C        ARTIFICIAL BOUNDARY
C
C        THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
      PARAMETER (NDIM = 2*IX+1)
C         THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16    P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX       LB, FACR
      COMPLEX       A( -IX:IX, -IX: IX)
      COMPLEX       AA( NDIM   , NDIM   ), BB( NDIM   , NDIM   )
      COMPLEX       AL( -40: 40)


      COMPLEX       LAND
      COMMON/SS/P, W
      COMMON/TT/KP,KF,INFA, THETAI,L, X, PI,
     1          EISP, OMEGA, T
      COMMON/CC/LB, FACR
      COMMON/FF/A
      COMMON/GG/GAMMA, BETTA
      COMMON/HH/MINN, MAXX
      COMMON/II/AA, BB
      COMMON/KK/IPVT
      COMMON/RN/RUNS
      COMMON/FEL/U,V,TP
      COMMON/SN/FREQ, SPD
C
      REAL     THETAI, INFA, KF, EISP, OMEGA, L, KP
      REAL     XI, X, T
      REAL     GAMMA(-40: 40), BETTA (-40:40), PI
      REAL     SPD, FREQ
      INTEGER  I, U, V, TP, MINN, MAXX, IPVT(NDIM), RUNS, Y
C
      Y = IY/2
      WRITE(2,75)
   75 FORMAT(1X,'  ')
      HEN = RUNS*DELTAT
C
      DO 10 M = MINN, MAXX,1
         AL(M) = CMPLX(0.0, 0.0)
           DO 30 I = -IX, IX, 1
C
C
```

```
              XI = I*H
              IF( ABS(I) .EQ. IX) THEN
                  DEL = .5
                  ELSE
                  DEL = 1.0
              ENDIF
          LAND = CEXP(-LB*Y*H*BETTA(M) + (LB*HEN))
          AL(M) = AL(M) + .5*LAND*H*DEL*CEXP(-LB*GAMMA(M)*XI)*P(I,Y,3)
 30     CONTINUE
 10   CONTINUE
      WRITE(2,32)
 32   FORMAT(1X,'ALPHA "N"S FOR RADIATING MODES')
      DO 111 M = MINN, MAXX
      WRITE(2,*)CABS(AL(M))
111   CONTINUE
      RETURN
      END
C
      SUBROUTINE CLOSE
C
C        THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
      PARAMETER (XYX= 40)
      PARAMETER (TNT= 1000)
      PARAMETER (NDIM = 2*IX+1)
C        THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16    P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMPLEX       LB, FACR
C        THE VALUES IN THE COMMON "BLOCKS" ARE VARIABLES
      COMMON/SS/P, W
      COMMON/TT/KP,KF,INFA, THETAI,L, X, PI,
     1           EISP, OMEGA ,T
      COMMON/CC/LB, FACR
      COMMON/HH/MINN, MAXX
      COMMON/GF/WGGRF,PGGRF
C
      REAL     KP, KF, INFA, THETAI, L, PI, EISP, OMEGA


      REAL     X, T
      REAL     XX( 0: 41), WPLOT( -IX: IX), ZZ(0:1000)
      REAL     PPLOS( -IX: IX), SMALL(1000)
      INTEGER  II, RUNS, SAM, IR
      PARAMETER (SAM = 1000)
      REAL     WGGRF( SAM ), PGGRF( SAM )
C
      DO 10 II = 0, XYX, 1
         XX(II) = II
   10 CONTINUE
C
      DO 11 IR = 0, TNT, 1
         ZZ(IR) = IR
   11 CONTINUE
C
      DO 30 I = 1,1000,1
         SMALL(I) = WGGRF(I)
   30 CONTINUE
      DO 35 I = -IX, IX, 1
         PPLOS(I) = CDABS(P(I,IY,3))
   35 CONTINUE
      DO 45 I = -IX, IX, 1
         WPLOT(I) = CDABS(W(I,3))
   45 CONTINUE
C
```

```
C
C
C
C
C
      CALL COMPRS
      CALL PLOTD(ZZ,SMALL,1000, .TRUE. ,'LINLIN','CONV FACTOR$',
     1          'CONVERGENCE GRAPH$','RUN NUMBER + 2000$',
     2          'CONVERGENCE VALUE$')
C
      CALL PLOTD(XX,PPLOS,41, .TRUE. ,'LINLIN','FLUID PRESSURE$',
     1          'PI/2  LENGTH=1.2  FREQ= 3750$','HORIZONTAL POSITION$',
     2          'SCALED PRESSURE$')
      CALL PLOTD(XX,WPLOT,41, .TRUE. ,'LINLIN','DISP AT 3750 HZ$',
     1          'SCALED LENGTH OF 1.2$','SCALED HORIZONTAL POSITION$',
     2          'SCALED DISPLACEMENT OF PLATE$')
      CALL DONEPL
      RETURN
      END
C
      SUBROUTINE FINN
C
C        THIS SUBROUTINE MAKES SURE WE HAVE REACHED THE STEADY STATE
C        SOLUTION OF THE PROBLEM, IE 'EQUALIBRIUM'
C        THE VALUES IN THE PARAMETER STATEMENTS ARE CONSTANTS
C
      PARAMETER (IX = 20, IY = 80)
      PARAMETER (H = .025, DELTAT = .0325)
C         THE FOLLOWING ARE COMPLEX ARRAYS
      COMPLEX*16   P( -IX: IX, 0: IY, 3), W( -IX: IX, 3)
      COMMON/SS/P, W
      COMMON/TT/KP,KF,INFA, THETAI, L, X, PI,
     1          EISP, OMEGA ,T
      COMMON/EE/WGTWO, WGTHE, WGTOT, CON
      COMMON/PT/PGTWO, PGTHE, PGTOT
      COMMON/RN/RUNS
      COMMON/GF/WGGRF,PGGRF
C
      REAL    KP, L, OMEGA, EISP, KF, INFA, PI, THETAI
      REAL    X, T
      REAL*8  WGTWO,WGTHE,WGTOT,CON
      REAL*8  PGTWO,PGTHE,PGTOT
      INTEGER I, RUNS, SAM
      PARAMETER (SAM = 1000)
      REAL     WGGRF( SAM ), PGGRF( SAM )
C
      DO  34 I =  -IX+2, IX-2, 1

        WGTWO  =   W(I,2)*DCONJG(W(I,2))+ WGTWO
        WGTHE  =   W(I,3)*DCONJG(W(I,3))+ WGTHE
        WGTOT  =   (WGTHE - WGTWO) / WGTHE
   34 CONTINUE
C
      DO  44 I =  -IX  , IX  , 1
        PGTWO  =   P(I,IY,2)*DCONJG(P(I,IY,2))+ PGTWO
        PGTHE  =   P(I,IY,3)*DCONJG(P(I,IY,3))+ PGTHE
        PGTOT  =   (PGTHE - PGTWO) / PGTHE
   44 CONTINUE
C
       IF (RUNS.GT. 1999) THEN
       WGGRF(RUNS-1999) = WGTOT
       PGGRF(RUNS-1999) = PGTOT
       ENDIF
C
C      WRITE(2,88)
C 88   FORMAT(1X,'WGGRF,PGGRF IN FINN')
C     WRITE(2,*) WGGRF(RUNS-1999)
C     WRITE(2,*) PGGRF(RUNS-1999)
      RETURN
      END
```

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